LESSON 10

Length of fin

Determination of fin length is an important step in design of a fin once it's cross sectional area has been determined. Longer the fin, larger is the surface area and higher is the heat transfer. However, it is not necessary that the fin should be infinitely long for maximum heat transfer. It has been observed that the temperature drops along the fin exponentially and reaches environmental temperature at some point along the fin length. Beyond this point, length of fin does not contribute to the heat transfer, therefore designing extra long fin results in material waste, excessive weight and increased size at increased cost without any benefit in return.

In order to determine the proper length of the fin, heat transfer from a fin of finite length is compared with that of infinite length.

$$\frac{Q_{finite}}{Q_{\inf inite}} = \frac{kA_c m\theta_0 \tanh mL}{kA_c m\theta_o} = \tanh mL \qquad (1)$$

Values of hyperbolic function of tanh mL are calculated for some values of mL and it has been found that Q increases linearly initially with increase in the vaue of 'mL' but then reaches a plateau for an infinitely long fin for which mL = 5, i.e., $L = \frac{5}{m}$. A fin of length $L = \frac{5}{m}$ is considered as infinitely long. If length of fin is reduced from $\frac{5}{m}$ to $\frac{2.5}{m}$, there is only 1% reduction in heat transfer. If $L = \frac{1}{m}$, then the fin will transfer about 70% of the heat that can be transferred by an infinitely long fin. Therefore, the length of the fin should be varied from $L = \frac{1}{m}$ to $\frac{2.5}{m}$.

Effectiveness of fin

The purpose of use of fins is to enhance heat transfer from a surface. Performance of a fin is characterized by fin effectiveness, 'ε' and is defined as ratio of heat transfer from a finned surface to that of without a fin.

$$\varepsilon = \frac{Heat\,transfer\;from\,a\;finned\;surface}{Heat\,transfer\;from\,an\;unfinned\;surface} \tag{2}$$

Consider the case of a rectangular fin that is losing heat at the tip only. Its effectiveness is given expressed as

$$\in = \frac{kA_c m\theta_0 (\frac{mk\sinh mL + h\cosh mL}{h\sinh mL + mk\cosh mL})}{hA_c\theta_0}$$

$$\in = \frac{mk}{h}(\frac{mk\sinh mL + h\cosh mL}{h\sinh mL + mk\cosh mL}) \tag{3}$$

Divide numerator and denominator by $\cosh mL$

$$\in = \frac{mk}{h}(\frac{mk\tanh mL + h}{h\tanh mL + mk})$$

$$\in = \frac{mk}{h}(\frac{\frac{mk}{h}\tanh mL + 1}{\tanh mL + \frac{mk}{h}}) \tag{4}$$

We know, $m = \sqrt{\frac{Ph}{kA_c}} = \sqrt{\frac{2(w+2\delta)h}{kw2\delta}} = \sqrt{\frac{2wh}{kw2\delta}}$ $\{\because w \rangle\rangle 2\delta\}$

$$\therefore m = \sqrt{\frac{h}{k\delta}} \tag{5}$$

Biot Number, $B_i = \frac{h\delta}{k} = \frac{\delta/k}{1/h} = \frac{\text{Internal resistance of fin material}}{\text{External resistance of fluid on fin surface}}$

(6)

$$\text{Now, } \frac{mk}{h} = \sqrt{\frac{h}{k\delta}}\frac{k}{h} = \sqrt{\frac{hk^2}{k\delta h^2}} = \sqrt{\frac{k}{h\delta}} = \frac{1}{\sqrt{B_i}} \tag{7}$$

$$\text{Also, } mL = \sqrt{\frac{h}{k\delta}}L \times \frac{\delta}{\delta} = \sqrt{\frac{h\delta^2}{k\delta}}\frac{L}{\delta} = \sqrt{\frac{h\delta}{k}}\overline{L} = \sqrt{B_i}\,\overline{L} \tag{8}$$

{where $\frac{L}{\delta} = \bar{L}$ }

Using equations (7) and (8) in equation (4),we get

$$\in = \frac{1}{\sqrt{B_i}} \left(\frac{\frac{1}{\sqrt{B_i}} \tanh \sqrt{B_i}\,\bar{L} + 1}{\tanh \sqrt{B_i}\,\bar{L} + \frac{1}{\sqrt{B_i}}} \right) \tag{9}$$

Equation (9) gives the expression of effectiveness of a fin in terms of Biot Number, B_i. If $B_i = 1$, there is no use of putting the fin as heat transfer rate will remain the same. If $B_i \rangle 1, \in \langle 1$, heat transfer rate decreases with the addition of fin as it acts as an insulator. If $B_i \langle 1, \in \rangle 1$, then this is desirable because heat transfer rate increases with the use of a fin.

Efficiency of fin

It is generally assumed that bond between a fin and surface, to which fin has been attached, is perfect and it does not offer any thermal resistance. Therefore, temperature at the base of the fin is maximum and equal to the surface temperature. Heat transfer from the base of fin to surrounding fluid will be the maximum as temperature difference is maximum at the base of fin and is expressed as

$$Q_{max} = hA_{fin}\,(T_o - T_a) = h\,PL\,(T_o - T_a) \tag{10}$$

However, due to conductive resistance of fin material, temperature decreases along length of fin, hence, heat transfer rate decreases due to decrease in temperature difference. To account for this decrease in heat transfer, fin efficiency is defined as

$$\eta_f = \frac{Actual\ heat\ transfer\ from\ fin\ surface}{Maximum\ heat\ transfer\ if\ entire\ fin\ surface\ is\ ma\text{int}\ ained\ at\ base\ temperature}$$

If fin tip is insulated, efficiency is expressed as

$$\eta_f = \frac{kA_c m\theta_o \tanh mL}{hPL\theta_o}$$

$$\eta_f = \frac{kA_c m \tanh mL}{hPL} \tag{11}$$

For a straight rectangular fin, A_c = w x 2δ and P= 2(w+2δ) as 2δ<< w, P=2w. Equation (11) becomes

$$\eta_f = \frac{k\,w \times 2\delta m \tanh mL}{h \times 2wL}$$

$$\eta_f = \frac{mk\,\delta \tanh mL}{h\,L} \tag{12}$$

As $$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h \times 2w}{kw \times 2\delta}} = \sqrt{\frac{h}{k\delta}} \tag{13}$$

Substituting the value of 'm' from equation (13) in equation (12), we get

$$\eta_f = \sqrt{\frac{hk^2\delta^2}{k\,\delta h^2 L^2}} \tanh mL$$

$$\eta_f = \frac{1}{L}\sqrt{\frac{k\delta}{h}} \tanh mL$$

Efficiency of most of the fins generally used is above 90 percent.

SOLVED EXAMPLES ON EXTENDED SURFACES

Ex. 10.1 A solid pipe of diameter 5 cm and length 15 cm is attached to a tank containing hot oil due to which temperature of the end of the pipe connected with tank is 250 ºC. The pipe is surrounded by a fluid maintained at 25 ºC and its thermal conductivity is 60 W/m-K. Determine the heat loss per hour from the pipe if convective heat transfer coefficient is 25 W/m²- K and heat loss at the free end of the pipe is neglected.

Solution:

Given: Diameter of pipe, d = 5 cm = 0.05 m, Length of pipe, L = 15 cm = 0.15 m

Temperature at base of fin, T_o= 250 °C, Temperature of Surroundings, T_a= 25°C,

Thermal conductivity, k = 60 W/m-K, Convective heat transfer coefficient, h = 25 W/m²-K

Area of cross-section, A_c = (π/4) d^2= 0.7855 X 0.05^2 = 0.00196 m^2

Perimeter, P = πd = 3.142 X 0.05 =0.1571 m

To determine: i) Heat loss per hour from pipe if heat loss from free end is neglected,

$$Q = kmA_c\,\theta_o\, tanhmL$$

$$Where\ m = \sqrt{\frac{Ph}{kA_c}} = \sqrt{\frac{0.1571 \times 25}{60 \times 0.00196}} = 5.78\ \text{m}^{-1}$$

$$\theta_o = T_o - T_a = 250 - 25 = 225$$

$$Therefore, Q = 60 \times 5.78 \times 0.00196 \times 225 \times tanh(5.78 \times 0.15) = 107.03\ W = 107.03\ \frac{J}{Sec}$$

$$= 385.30\ kJ/Hour$$

Ex 10.2 Fins attached to an electronic device maintain its temperature at 50 ºC by removing 60 mW of energy. The fin has a cross section of (0.8mm × 0.8mm) and is 2 cm long. Determine the number of fins required if there is no loss of heat from the fin end and thermal conductivity of fin material is 100 W/m-K. Temperature of surrounding air is 30 ^{o}C and convective heat transfer coefficient between fin and air is 8.5 W/m ^{2} - K.

Solution:

Given: Width of single fin, b = 0.8 mm = 0.0008 m,

Thickness of single fin, δ = 0.8 mm = 0.0008 m,

Perimeter, P = 2(b+δ) = 2 (0.0008+0.0008) = 0.0032 m

Area of cross-section, A_c = b X δ = 0.0008 X0.0008 = 6.4 X 10^{-7} m^2

Length of a single fin, L = 2 cm = 0.02 m

Temperature at base of fin, T_o= 50 °C, Temperature of Surroundings, T_a= 30°C,

Temperature difference at the base of fin, θ_o =T_o-T_a = 50-30 =20

Thermal conductivity, k = 150 W/m-K, Convective heat transfer coefficient, h = 8.5 W/m^2-K

Amount of heat to be removed by 'n' of fins, q = 60 mW = 60 X 10^{-3} W

To determine: i) Number of fins required if heat loss from fin end is neglected

$$n = \frac{Amount\ of\ heat\ to\ be\ removed\ by\ 'n'fins}{Heat\ removed\ by\ a\ single\ fin} = \frac{q}{Q}$$

$$Where, Q = \boldsymbol{kmA_c\,\theta_o\,tanhmL}$$

$$\boldsymbol{m} = \sqrt{\frac{\boldsymbol{Ph}}{\boldsymbol{kA_c}}} = \sqrt{\frac{\mathbf{0.0032\ \times 8.5}}{\mathbf{150 \times 6.4 \times 10^{-7}}}} = \mathbf{16.83\ m^{-1}}$$

$$Q = \mathbf{150 \times 16.83 \times 6.4 \times 10^{-7} \times 20 \times \boldsymbol{tanh}(16.83 \times 0.02)} = \mathbf{0.0105}\ \boldsymbol{W} = \mathbf{10.5}\ \boldsymbol{mW}$$

$$\mathbf{Therefore, n ==} \frac{\mathbf{60\ \times 10^{-3}}}{\mathbf{10.5\ \times 10^{-3}}} = \mathbf{5.71} \approx \mathbf{6}$$

Ex 10.3 Base of an infinitely long rod of 5 cm diameter is maintained at a temperature of 125 °C. Determine heat loss from the rod if ambient temperature is 35 °C. Thermal conductivity of the rod material is 45 W/m-K and convective heat transfer coefficient is 35 W/m ^{2} - K.

Solution:

Given: Diameter of rod, d = 5 cm = 0.05 m, Perimeter, P = πd = 3.142 X 0.05 =0.1571 m

Temperature of rod, T_o= 125 °C, Temperature of Surroundings, T_a= 35°C,

Thermal conductivity, k = 45 W/m-K, Convective heat transfer coefficient, h = 35 W/m^2-K

Area of cross-section, A_c = (π/4) d^2= 0.7855 X 0.05^2 = 0.00196 m^2

To determine: i) Heat loss from infinitely long rod,

$$\boldsymbol{Q = kmA_c\,\theta_o}$$

$$\boldsymbol{Where,}\qquad \boldsymbol{m} = \sqrt{\frac{\boldsymbol{Ph}}{\boldsymbol{kA_c}}} = \sqrt{\frac{\mathbf{0.1571\ \times 35}}{\mathbf{45 \times 0.00196}}} = \mathbf{7.89\ m^{-1}}$$

$$\boldsymbol{\theta_o\ = T_o - T_a} = \mathbf{125 - 35 = 90}$$

$$\boldsymbol{Therefore, Q} = \mathbf{45 \times 7.89 \times 0.00196 \times 90 = 62.67}\ \boldsymbol{W}$$

Ex 10.4 Temperature difference of 60 °C is maintained at the base of a fin of thickness 3 mm, height 40 mm and width 80 cm. Determine the heat transferred from fin when it is losing heat only at tip and temperature difference at the tip of the fin when ambient temperature is equal to 20 °C, thermal conductivity of fin material is 50 W/m-deg and convective heat transfer coefficient as 20 W/m²-deg.

Solution:

Given: Width of single fin, b = 80 cm = 0.8 m, Thickness of single fin, δ = 3 mm = 0.003 m,

Perimeter, P = 2(b+δ) = 2 (0.8+0.003) = 1.606 m,

Area of cross-section, A_c = b X δ = 0.8 X 0.003 = 0.0024 m^2,

Height or Length of fin, L = 40 mm = 0.04 m

Temperature difference at the base of fin, θ_o =60 °C, Thermal conductivity, k = 50 W/m-deg,

Convective heat transfer coefficient, h = 20 W/m^2-deg

To determine: i) Heat loss from tip of the fin

ii) Temperature difference at tip of fin

i) Heat loss from tip of the fin

$$Q = kmA_c\,\theta_o \frac{(mk \sinh mL + h\cosh mL)}{(h\sinh mL + mk\cosh mL)}$$

$$\boldsymbol{Where, m} = \sqrt{\frac{\boldsymbol{Ph}}{\boldsymbol{kA_c}}} = \sqrt{\frac{\mathbf{1.606 \times 20}}{\mathbf{50 \times 0.0024}}} = \mathbf{16.36\ m^{-1}}$$

$$\text{Therefore, } Q = 50 \times 16.36 \times 0.0024 \times 60 \times \frac{\mathbf{16.36 \times 50 \times sinh(16.36 \times 0.04) + 20 \times cosh(16.36 \times 0.04)}}{\mathbf{20 \times sinh(16.36 \times 0.04) + 16.36 \times 50 \times cosh(16.36 \times 0.04)}} = \mathbf{69.58\ W}$$

ii) Temperature difference at tip of fin, θ

$$\theta = \theta_o \left[coshmx - \frac{(mk \sinh mL + h\cosh mL)}{(h\sinh mL + mk\cosh mL)} sinhmx \right]$$

At tip of fin, x=L,

$$\text{Therefore, } \theta = 60 \times \left[\cosh(0.6544) - \frac{\mathbf{16.36 \times 50 \times sinh(0.6544) + 20 \times cosh(0.6544)}}{\mathbf{20 \times sinh(0.6544) + 16.36 \times 50 \times cosh(0.6544)}} \times \sinh(0.6544) \right]$$

$$\theta = 60 \times \left[1.22 - \frac{818\times0.701+20\times1.22}{20\times0.701+50\times1.22} \times 0.701\right]$$

$$\theta = 60 \times (1.22 - 0.4140) = \mathbf{48.35\ ^oC}$$

Ex 10. 5 A fin of 5 mm square section with thermal conductivity of 90 W/m-deg has been attached to a furnace wall at 250°C and is exposed to air at 40°C with convection coefficient of 50 W/m²-K. Determine the length of the rod so that 90 % of total heat conducted from an infinitely long fin is dissipated by convection.

Solution:

Given: Width of single fin, b = 5 mm = 0.005 m, Thickness of single fin, δ = 5 mm = 0.005 m,

Perimeter, P = 2(b+δ) = 2 (0.005+0.005) = 0.02 m,

Area of cross-section, A_c = b X δ = 0.005 X 0.005 = 2.5 X 10^{-5} m^2,

Temperature at the base of fin, T_o= 250 °C, Temperature of air, T_a= 40°C,

Temperature difference at the base of fin, θ_o = T_o - T_a = 250-40 = 210

Thermal conductivity, k = 90 W/m-deg, Convective heat transfer coefficient, h = 50 W/m²-deg

To determine: i) Length of fin required for dissipation of 90% heat conducted from the fin

Heat dissipation for an infinity long fin is

$$Q = kA_c m(T_0 - T_a)$$

$$Wherer, m = \sqrt{\frac{Ph}{kA_c}} = \sqrt{\frac{0.02 \times 50}{90 \times 2.5 \times 10^{-5}}} = \mathbf{21.08\ m^{-1}}$$

$$\text{Therefore, } Q = 90 \times 2.5 \times 10^{-5} \times 21.08 \times 210 = \mathbf{9.96\ W}$$

Length of fin is to be determined at which 90% of 'Q' is dissipated which is **8.964 W**.

For an infinitely long fin, the temperature distribution is given as

$$\theta = \theta_o\,(coshmx - cothmL \times sinhmx)$$

As L→ ∞, CothmL→1, Therefore, $\theta = \theta_o\,(coshmx - sinhmx) = \theta_o\, e^{-mx}$

$$As\ e^{-mx} = coshmx - sinhmx$$

$$\text{Therefore, } \frac{\theta}{\theta_0} = \frac{T - T_a}{T_0 - T_a} = e^{-mx}$$

Let at fin length of 8 mm, 90% of heat is dissipated.

$$mx = 21.08 \times 0.008 = 0.16864$$

$$\frac{T-40}{250-40} = e^{-0.16864} = 0.8448$$

$$T = 0.8448 \times 210 + 40 = \mathbf{217.41}℃$$

Heat convected up to any length is given by the difference of total heat conducted by infinitely long fin and heat conducted at given length of the fin.

∴ Heat convected up to 0.08 m length

$$= 9.96 - k\,A_c m(T_{0.08} - T_a)$$

$$= 9.96 - 90 \times 2.5 \times 10^{-5} \times 21.08 \times (217.41 - 40) = 1.545\,W$$

Which is $\frac{1.545}{9.96} \times 100 = 15.51\,\%\ of\ total\ heat\ dissipation.$

Let at fin length of 109 mm, 90% of heat is dissipated.

$$mx = 21.08 \times 0.109 = 2.2977$$

$$\frac{T-40}{250-40} = e^{-2.2977} = 0.1004$$

$$T = 0.1004 \times 210 + 40 = \mathbf{61.10}℃$$

Heat convected up to 0.109 m length

$$= 9.96 - k\,A_c m(T_{0.109} - T_a)$$

$$= 9.96 - 90 \times 2.5 \times 10^{-5} \times 21.08 \times (61.10 - 40) = 8.96\,W$$

Which is $\frac{8.96}{9.96} \times 100 = 89.96 \approx 90\%\ of\ total\ heat\ dissipation.$

Therefore, most of the heat will be dissipated for a fin length of 109 mm and extending the fin beyond this length will not be economically viable.

Ex 10.6 A fin with a base temperature of 250 ºC is surrounded by air at 25 ºC. Thermal conductivity of fin is 150 W/m-deg and convective heat transfer coefficient is 50 W/m^2-deg. If fin has cross-sectional area 1.5 cm^2, perimeter 6 cm and heat loss from tip of fin is neglected, determine which of the following arrangement of pin fins will give higher heat transfer rate?

4 fins of 10 cm length
10 fins of 5 cm length

Solution:

Given: Perimeter, P = 6 cm= 0.06 m, Area of cross-section, A_c = 1.5 cm^2 = 1.5 X 10^{-4} m^2,
Temperature at the base of fin, T_o= 250 °C, Temperature of air, T_a= 25°C,
Temperature difference at the base of fin, θ_o = T_o - T_a = 250-25 = 225
Thermal conductivity, k = 150 W/m-deg,
Convective heat transfer coefficient, h = 50 W/m^2-deg

To determine: i) Heat transfer from a fin arrangement with insulated tip

$$Q = n\,k\,A_c\,m(T_o - T_a)\tanh ml$$

$$\text{Where } m = \sqrt{\frac{Ph}{kA_c}} = \sqrt{\frac{0.06\times50}{150\times1.5\times10^{-4}}} = \mathbf{11.54\ m^{-1}}$$

Case I n = 4 and L = 10cm = 0.1 m
mL = 11.54 X 0.5 = 1.154
Q_1 = 4× [150×1.5×10^{-4}×11.54×(250-25) tanh (1.154)]
= **191.40 W**

Case II n = 10 and L = 5 cm = 0.05 m
mL = 11.54×0.05=0.577
Q_2 = 10× [150×1.5×10^{-4}×11.54×(250-25) tanh (0.577)]
= **303.28 W**

The second arrangement should be preferred as it gives higher rate of heat transfer.

Ex 10.7 Twenty five fins have been attached to a pipe of diameter 6 cm and length 1.25 m to enhance the heat transfer from the pipe. A single fin of length 5 cm has a rectangular cross sectional area of 0.0032 m^2 and perimeter of 1.3 m. The pipe is carrying hot fluid due to which temperature at the base of fins is 70 °C. Temperature of surrounding air is 20 °C and convective heat transfer coefficient of the fins as well as the tube surface with the surrounding air is 12.5 W/m^2-K. Determine the heat dissipated from the pipe with and without fins if thermal conductivity of the fin material is 35 W/m-K and fin tip is assumed to be insulated.

Solution: Given: Diameter of pipe, d = 6 cm = 0.06 m, Length of pipe, L_p = 1.25 m,

Surface area of the pipe, $A_p = \pi \times d \times L_p = 3.142 \times 0.06 \times 1.25 = 0.235\ m^2$

Perimeter, P = 1.3 m, Area of cross-section, A_c = 0.0032 m^2,

Length of fin = 5 cm = 0.05 m, Number of fins, n = 25

Temperature at base of fin, T_o= 70 °C, Temperature of Surroundings, T_a= 40°C,

Temperature difference at the base of fin, θ_o = T_o - T_a = 90-20 = 70

Thermal conductivity, k = 35 W/m-K, Convective heat transfer coefficient, h = 12.5 W/m^2-K

To determine: i) Heat loss from the pipe without fins

ii) Heat loss from the pipe with fins as tip of fins is assumed to be insulated

i) **Heat loss from the pipe without fins**

$$Q_1 = h\,A_p\,(T_0 - T_a) = 12.5 \times 0.235 \times (70 - 20) = \mathbf{146.87\ W}$$

ii) **Heat loss from the pipe with fins as tip of fins is assumed to be insulated**

$$Q_2 = Q_{bare} + Q_{Fins} = hA_{bare}(T_o - T_a) + Q_{Fins}$$

$$Where,\ A_{bare} = Surface\ Area\ of\ pipe - Area\ of\ twenty\ five\ fins$$
$$= A_p - 25 \times A_f = 0.235 - 25 \times 0.0032 = 0.155\,m^2$$

$$Therefore, Q_{bare} = 12.5 \times 0.155 \times (70 - 20) = \mathbf{96.87\ W}$$

$$and\ Q_{Fins} = n \times k\,A_c\,m(T_0 - T_a)\tanh mL$$

$$m = \sqrt{\frac{Ph}{kA_c}} = \sqrt{\frac{1.3 \times 12.5}{35 \times 0.0032}} = 12.04\,m^{-1}$$

$$Therefore, Q_{fins} = 25 \times 35 \times 0.0032 \times 12.04 \times 50 \times \tanh(12.04 \times 0.05)$$
$$= \mathbf{907.64\ W}$$

∴ Heat flow rate from the pipe surface when fins are fitted,

$Q_2 = Q_{bare} + Q_{fins} = \mathbf{96.87 + 907.64 = 1004.51\ W}$

Ex 10.8 A horizontal pin fin of 15 mm diameter and 50 cm long is connected to a combustion chamber in such a way that its base is maintained at 800° C. Temperature and convection heat transfer coefficient associated with the fluid surrounding the fin are 40°C and 80 W/m^2-deg. If thermal conductivity of fin material is 90 W/m-deg determine the temperature of fin at distance of 80 mm from the base under the conditions: (a) the fin is very long (b) the heat flow through the end of the fin is negligible and (c) heat is transferred to the surroundings from the end of fin only.

Solution:

Given: Diameter of pin fin, d = 15 mm = 0.015 m, Length of pin fin, L = 50 cm =0.5 m

Area of cross-section, $A_c = (\pi/4)\,d^2 = 0.7855 \times 0.015^2 = 1.76 \times 10^{-4}\ m^2$

Perimeter, $P = \pi d = 3.142 \times 0.015 = 0.0471$ m

Temperature at base of fin, T_o= 800 °C, Temperature of Surroundings, T_a= 40°C,

Thermal conductivity, k = 90 W/m-K, Convective heat transfer coefficient, h = 80 W/m²-K

To determine: Temperature of fin at a distance of 80 mm from base under following conditions

i) **The fin is very long**

ii) **Heat flow through the end of the fin is negligible**

iii) **Heat is transferred to the surroundings from the end of fin only**

$$m = \sqrt{\frac{Ph}{kA_c}} = \sqrt{\frac{0.0471\times 80}{90\times 1.76\times 10^{-4}}} = \mathbf{15.42\ m^{-1}}$$

x= 80 mm = 0.08 m

i) **The fin is very long**

$$\frac{\theta}{\theta_0} = \frac{T_x - T_a}{T_0 - T_a} = e^{-mx}$$

Therefore, temperature T_x at $x = 0.08m$ is

$$T_x = T_a + (T_0 - T_a) \times e^{-mx}$$

$$= 40 + (800\text{-}40) \times e^{-15.42\times 0.08} = \mathbf{261.34\ ^oC}$$

ii) **Heat flow through the end of the fin is negligible**

$$\frac{\theta}{\theta_0} = \frac{T_x - T_a}{T_0 - T} = \frac{\cosh m(L-x)}{\cosh mL}$$

Therefore, temperature T_x at $x = 0.08m$ is

$$T_x = T_a + (T_0 - T_a)\frac{\cosh m(L-x)}{\cosh mL}$$

$$= 40 + (800 - 40) \times \frac{\cosh(15.42(0.5 - 0.08))}{\cosh(15.42 \times 0.5)}$$

$$=40 + 760\times 0.291 = \mathbf{261.34\ °C}$$

iii) **Heat is transferred to the surroundings from the end of fin only**

$$\frac{\theta}{\theta_0} = \frac{T_x - T_a}{T_0 - T_a} = \frac{\cosh m(L-x) + \frac{h}{mk}\sinh \mathrm{m}(L-x)}{\cosh \mathrm{mL} + \frac{h}{mk}\sinh \mathrm{m}\, L}$$

Therefore, temperature T_x at $x = 0.08\, m$ is

$$T_x = T_a + (T_0 - T_a) \times \frac{\cosh m(L-x) + \frac{h}{mk} \sinh \mathrm{m}(L-x)}{\cosh \mathrm{mL} + \frac{h}{mk} \sinh \mathrm{m}\, L}$$

$$= 40 + (800 - 40) \times \frac{\cosh(15.42(0.5 - 0.08)) + \frac{80}{15.42 \times 90} \sinh(15.42(0.5 - 0.08))}{\cosh(15.42 \times 0.5) + \frac{80}{15.42 \times 90} \sinh(15.42 \times 0.5)}$$

$$= 40 + 760 \times \left(\frac{324.81 + 0.0576 \times 324.81}{1115.27 + 0.0576 \times 1115.27}\right)$$

$= 40+760\times0.195 =$ **188.18°C**