LESSON- 13

Empirical Relations for Free and Forced Convection

1. Free Convection:

In case of free convection, heat transfer coefficient or Nusselt is expressed as

$$Nu_a = h\,L/k = f\,(Gr\,,\,Pr)$$

Where

Nu_a is average Nusselt Number
Gr is Grashoff number
Pr is Prandtl Number

A) For Vertical Plates and Cylinders

$$Nu_a = \frac{h_a L}{K} = 0.53\,(Gr\,Pr)^{\frac{1}{4}} \qquad \text{for } (Gr\,Pr) < 10^5$$

$$Nu_a = \frac{h_a L}{K} = 0.56\,(Gr\,Pr)^{\frac{1}{4}} \qquad \text{for } 10^5 < (Gr\,Pr) < 10^8$$

$$Nu_a = \frac{h_a L}{K} = 0.13\,(Gr\,Pr)^{\frac{1}{3}} \qquad \text{for } 10^8 < (Gr\,Pr) < 10^{12}$$

where
L is Characteristic length and it is the height of the plate or cylinder
h_a is average heat transfer coefficient.
Gr is Grashoff Number

B) Horizontal Cylinders

$$Nu_a = \frac{h_a L}{K} = 1.1\,(Gr\,Pr)^{\frac{1}{6}} \qquad \text{for } \frac{1}{10} < (Gr\,Pr) < 10^4$$

$$Nu_a = \frac{h_a L}{K} = 0.53\,(Gr\,Pr)^{\frac{1}{4}} \qquad \text{for } 10^4 < (Gr\,Pr) < 10^9$$

$$Nu_a = \frac{h_a L}{K} = 0.13\,(Gr\,Pr)^{\frac{1}{3}} \qquad \text{for } 10^9 < (Gr\,Pr) < 10^{12}$$

Where
L is Characteristic length and in this case it is the diameter of the cylinder
h_a is average heat transfer coefficient.
Gr is Grashoff Number

C) Horizontal Square or Circular Plates

- For horizontal hot surface facing upward or cold surface facing downward.

$$Nu_a = \frac{h_a L}{K} = 0.71\,(Gr\,Pr)^{\frac{1}{4}} \qquad \text{for } 10^3 < (Gr\,Pr) < 10^9$$

$$Nu_a = \frac{h_a L}{K} = 0.17\,(Gr\,Pr)^{\frac{1}{3}} \qquad \text{for } (Gr\,Pr) > 10^9$$

- For horizontal hot surface facing downward or cold surface facing upward.

$$Nu_a = \frac{h_a L}{K} = 0.35\,(Gr\,Pr)^{\frac{1}{4}} \qquad \text{for } 10^3 < (Gr\,Pr) < 10^9$$

$$Nu_a = \frac{h_a L}{K} = 0.08\,(Gr\,Pr)^{\frac{1}{3}} \qquad \text{for } (Gr\,Pr) > 10^9$$

Where

L is Characteristic length and in case of square plate it is the side of the plate

L is Characteristic length and in case of circular plate it is the diameter

h_a is average heat transfer coefficient.

Gr is Grashoff Number

The properties of the fluid should be calculated at the temperature $\frac{T_s + T_f}{2}$

Where T_s = Plate surface temperature T_f = Fluid temperature.

2. Forced Convection

In case of forced convection, heat transfer coefficient or Nusselt is expressed as

$$Nu_x = f\,(x^*, Re_x, Pr)$$

Subscript 'x' has been added to emphasize our interest in conditions at a particular location on the surface.

Where

Nu_x is local Nusselt Number

Re_x is local Reynolds Number

$x^* = \frac{x}{L}$ is dimensionless distance

$$Nu_a = f\,(Re_L, Pr)$$

Subscript 'a' indicates an average distance from $x^* = 0$ to the location of interest.

Where,

Nu_a is average Nusselt Number

Re_L is Reynolds number at the location of interest

(A) Flow of fluid over a flat surface at constant temperature

- For laminar flow over flat plate which is valid for $Re_L < 5 \times 10^5$.

$$Nu_a = \frac{h_a L}{K} = 0.664 \quad Re_L^{\frac{1}{2}} \quad Pr^{\frac{1}{3}}$$

$$Nu_x = \frac{h_x X}{K} = 0.332 \quad Re_x^{\frac{1}{2}} \quad Pr^{\frac{1}{3}}$$

where h_a is average heat transfer coefficient.

h_x is the local heat transfer coefficient.

$$h_a = \frac{1}{L}\int_0^L h_x dx$$

$Nu_a = 2\ Nu_x$ and $h_a = 2\ h_x$

- If the flow condition on the flat plate is partly laminar and partly turbulent then for

  i) Only Laminar region

$$Nu_a = \frac{h_a L}{K} = 0.664\ Re_L^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$$

$$Nu_x = \frac{h_x x}{K} = 0.332\ Re_x^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$$

$$h_a = \frac{1}{L}\int_0^L h_x dx$$

where, h_a is average heat transfer coefficient.
h_x is the local heat transfer coefficient.

ii) Only Turbulent region, which is valid for $Re_L > 5 \times 10^5$,

$$Nu_a = \frac{h_a L}{K} = 0.037\ Re_L^{0.8}\ Pr^{\frac{1}{3}}$$ Which is valid for $5 \times 10^5 < Re_L < 10^7$

$$Nu_x = \frac{h_x x}{K} = 0.0296\ Re_x^{0.8}\ Pr^{\frac{1}{3}}$$ Which is valid for $5 \times 10^5 < Re_L < 10^7$

$$h_a = \frac{1}{L}\int_0^L h_x dx$$

Where h_a is average heat transfer coefficient.
h_x is the local heat transfer coefficient.

iii) Both Laminar and Turbulent region (mixed flow)

$$Nu_a = \frac{h_a L}{K} = \left[0.037\ Re_L^{0.8} - 871\right] Pr^{\frac{1}{3}}$$ which is valid for $5 \times 10^5 < Re_L < 10^8$

The properties of the fluid should be calculated at the temperature $\frac{T_S + T_f}{2}$

Where T_S is plate surface temperature
T_f is fluid temperature

(B) Fluid is flowing inside the tube or through the annulus

$$Nu_a = \frac{h_a L}{K} = 0.023\ Re_D^{0.8}\ Pr^{0.4}$$

Where L is Characteristic length and in this case, it is the diameter of the pipe

where $2300 < Re_D < 12 \times 10^4$

and $0.7 < Pr < 120$, and $\frac{L}{D} < 60$

The properties of the fluid should be taken at the mean temperature of the fluid T_f defined as: $T_f = \frac{T_s + T_m}{2}$ where $T_m = \frac{T_i + T_o}{2}$

Where T_i and T_o are the inlet and outlet temperatures of the fluid and T_s is surface temperature of the tube.

Characteristic Length or Equivalent Diameter (L_c or D_e):

Equivalent diameter is usually expressed by the following equation

$$D_e = \frac{4A_c}{P} = \frac{4\,\frac{\pi}{4}D^2}{\pi D} = D$$

Where A_c = Cross-sectional Area and P = Perimeter.

So for circular tube D_e = D (inner diameter of the tube). The equivalent diameter is also known as characteristic length. The characteristic lengths of a few geometries are given below.

1) The fluid is flowing through a rectangular duct as shown in Figure 1, then

$$L_c = \frac{4A_c}{P} = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

if a = b, then

$$L_c = \frac{2a^2}{2a} = 2a$$

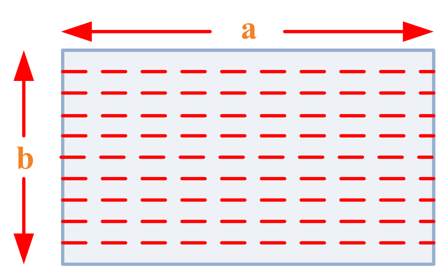

Figure 1

2) If the fluid is flowing through the annulus as shown in Figure 2, then

$$L_c = \frac{4A_c}{P} = \frac{4}{1} \times \frac{\frac{\pi}{4}\left(D^2 - d^2\right)}{1} \times \frac{1}{\pi\left(D+d\right)} = \left(D-d\right)$$

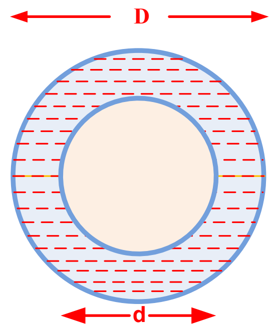

Figure 2

3) If the fluid is flowing through the annulus as shown in Figure 3, then

$$L_c = \frac{4A_c}{P} = \frac{4(a_1 b_1 - a_2 b_2)}{2[(a_1 + b_1) + (a_2 + b_2)]}$$

If $a_1 = b_1$ and $a_2 = b_2$, then

$$L_c = \frac{4A_c}{P} = \frac{4(a_1^2 - a_2^2)}{(2a_1 + 2a_2)} = (a_1 - a_2)$$

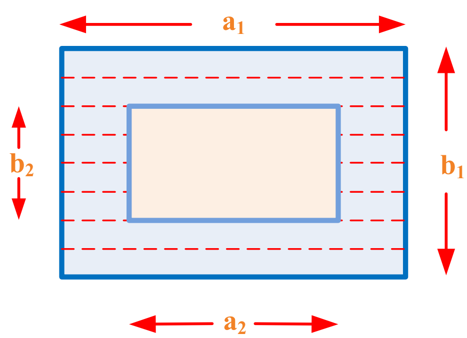

Figure 3

4) If the fluid is flowing through the annulus as shown in Figure 4, then

$$L_c = \frac{4A_c}{P} = \frac{4\left(a\ b - \frac{\pi}{4}d^2\right)}{2(a + b) + \pi d}$$

If $a = b$, then

$$L_c = \frac{4a^2}{4a + \pi d}$$

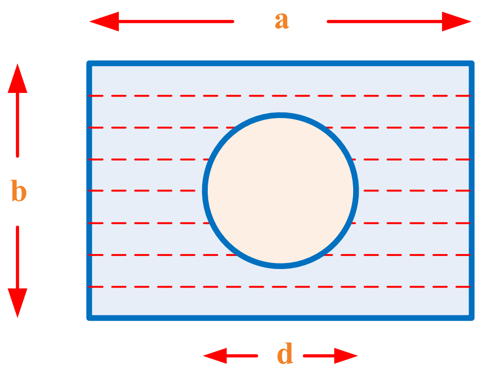

Figure 3

REVIEW QUESTIONS:

Q.1 Equivalent diameter or length, L_c is espressed as

a) 4 × Perimeter / Area of cross-section

b) Area of cross-section / Perimeter

c) Area of cross-section × Perimeter

d) 4 × Area of cross-section / Perimeter

Q.2 Equivalent diameter or length, L_c of a pipe is equal to its

a) Area of cross-section

b) Length

c) Diameter

d) Perimeter

Q.3 Mean fluid temperature at which properties of fluid are determined is equat to

a) Average of surface and fluid temperature

b) Averae of fluid temperature

c) Sum of independent and dependent varialbes

d) Differnce of independent and dependent variables

Q.4 If product of Grashoff and Prandtl numbers is less than 10^5, then

a) $Nu_a = \frac{h_a L}{K} = 0.56\ (Gr\ Pr)^{\frac{1}{4}}$

b) $Nu_a = \frac{h_a L}{K} = 0.13\ (Gr\ Pr)^{\frac{1}{3}}$

c) $Nu_a = \frac{h_a L}{K} = 0.18\ (Gr\ Pr)^{\frac{1}{2}}$

d) $Nu_a = \frac{h_a L}{K} = 0.53\ (Gr\ Pr)^{\frac{1}{4}}$