LESSON-17

SOLVED EXAMPLES ON CONVECTION

Ex. 17.1 **A pipe of 30 mm diameter and 2 m length is carrying hot gases due to which temperature at outer surface of the pipe is 500 ºC. In order to dissipate the heat, air at temperature 40° C is made to flow across the pipe with a velocity of 5 m/s. Determine the rate of heat transfer using the following relation.**

$$\mathbf{Nu = 0.6 \times (R_{ed})^{0.466} \left(\frac{T_s}{T_a}\right)^{0.12}}$$

Where T_s and T_a are the absolute temperatures of the pipe surface and air.

Take the following properties of air at $\left(\frac{40+500}{2}\right) = 270°C$

Thermal conductivity of air, k=5.23× 10^{-2}W/m − k

Kinematic viscosity of air, $\nu = 6.5 \times 10^{-5}\ m^2/s$.

Solution: **Given: Diameter of pipe, d=30 mm= 0.03 m, Length of pipe, L=2 m**

Temperature of pipe surface, T_s=500 ºC, Temperatue of air, T_a=40 ºC

Thermal conductivity of air, k=5.23× 10^{-2}W/m − k

Velocity of air, V = 5 m/sec

Kinematic viscosity of air, $\nu = 6.5 \times 10^{-5}\ m^2/s$.

To determine: i) Heat transfer from pipe surface to air

$$Q = hA(T_s - T_a)$$

Where, h is convective heat transfer coefficient and is given as

$$Nu = \frac{h \times D}{k} \quad or \quad h = \frac{Nu \times k}{D}$$

Nu is Nusselt number and is given as

$$Nu = 0.6 \times (R_{ed})^{0.466} \left(\frac{T_s}{T_a}\right)^{0.12}$$

$$Re = \frac{VD}{\nu} = \frac{5 \times 0.03}{6.5 \times 10^{-5}} = \mathbf{2308}$$

$$\frac{T_s}{T_a} = \left(\frac{500 + 273}{40 + 273}\right) = \mathbf{2.47}$$

$$\therefore \mathrm{Nu} = \frac{\mathrm{hd}}{\mathrm{K}} = 0.6 \times (2308)^{0.466}\,(2.47)^{0.12} = \mathbf{24.7}$$

$$\mathrm{h} = \frac{24.7 \times \mathrm{K}}{\mathrm{d}} = \frac{24.7 \times 5.23 \times 10^{-2}}{0.03} = 43.06\ \mathrm{W/m^2 - K}$$

The rate of heat transfer from pipe surface to air

$$\mathrm{Q} = \mathrm{hA}(\mathrm{T_s} - \mathrm{T_a})$$

$$= 43.06 \times (\pi \times 0\,.03 \times 2)(500 - 40) = \mathbf{3734\ W}$$

Ex 17.2 **Determine the heat transfer from a sphere of diameter 25 mm when its surface is maintained at a temperature of 175 ^{o}C by an heating element of 100 W capacity and temperature of surrounding air is 35 ^{o}C. The appropriate correlation for the convection coefficient is**

$\mathrm{Nu} = 0.60\ (\mathrm{Gr\ Pr})^{0.25}$

Thermo-physical properties of air at mean fluid temperatue are as

Thermal conductivity of air, k= $\mathbf{0.05\ W/m - deg}$

Kinematic viscosity of air, ν = 2.15$\times$ $\mathbf{10^{-5}\ m^2/s}$

Prandtl Number, Pr = 0.693

Solution: **Given: Diameter of sphere, d=25 mm= 0.025 m**

Temperature of sphere, T_s=175 ^{o}C, Temperatue of air, T_a=35 ^{o}C

Thermal conductivity of air, k=0.55 W/m-deg

Kinematic viscosity of air, $\nu = 2.15 \times 10^{-5}\ \mathrm{m^2/s}$.

At the mean film temperature, T_f = (275+35)/2=155°C, the thermo- physical properties for air are:

Kinematic viscosity, ν = 2.15$\times$ 10^{-5} m^2/s

Thermal conductivity, $\mathbf{k = 0.05 W/m - deg}$

Prandtl number, Pr = 0.693 ,

To determine: i) Heat transfer from sphere to air

$$\mathbf{Q = hA(T_s - T_a)}$$

$$Where, h\ is\ convective\ heat\ transfer\ coefficient\ and\ is\ given\ as$$

$$Nu = \frac{h \times D}{k}\ or\ h = \frac{Nu \times k}{D}$$

Nu is Nusselt number and is given as

$$\boldsymbol{Nu} = \mathbf{0.60} \times\ (\mathbf{Gr} \times\ \mathbf{Pr})^{\mathbf{0.25}}$$

Where, Gr is Grashoff number and is expressed as

$$Gr = \frac{L^3\rho^2\beta g\Delta T}{\mu^2} = \frac{L^3\beta g\Delta T}{\nu^2}$$

$$\beta = \frac{1}{T_f} = \frac{1}{273+105} = 2.64 \times 10^{-3} \text{per deg kelvin}$$

$$\text{Therefore, Gr} = \frac{(0.025)^3 \times 2.64\times10^{-3}\times9.81\times(175-35)}{(2.15\times10^{-5})^2} = \mathbf{40972/\ 122558.68}$$

Using the given correlation

$$Nu = \frac{hL}{k} = 0.60 \times (\mathbf{122558.68}$$

$\times$ 0.693)0.25 =10.24

$$h = \frac{0.025}{0.05} \times 0.60 \times (8.0 \times 10^5 \times 0.693)^{0.25} = 20.48\ \mathbf{W/m^2 - K}$$

Therefore, heat transfer rate is

$$Q = h\,A\,\Delta T = 20.48 \times \pi \times 0.025^2 \times (175 - 35) = \mathbf{5.6\ W}$$

Ex 17.3 **An electric heater has been intalled inside a square plate (25 cm X 25 cm) to maintain a temperature of 125 °C at the outer surfaces of the plate which are sorrounded by air at 30 °C. Determine the heat loss from the plate surfaces to air under the following conditions:**

a) the plate is kept vertical
b) the plate is kept horizontal

The following empirical correlations have been suggested:
Nu = 0.125 $(Gr\ Pr)^{0.33}$ for vertical position of plate, and
Nu = 0.72 $(Gr\ Pr)^{0.25}$ for upper surface of horizontal plate
= 0.35 $(Gr\ Pr)^{0.25}$ for lower surface of horizontal plate

The thermo-physical properties of air at mean temperature are as:

Density, $\rho = 1.06\ kg/m^3$, Thermal conductivity, $k = 0.028\ W/m - deg$

Specific heat, $C_P = 1.008\ kJ/kg - k$ and Knematic viscosity, $\nu = 18.97 \times 10^{-6}\ m^2/s$

Solution: Given: Temperature of surface of plate, T_s=125 °C, Temperatue of air, T_a=30 °C
Density of air, ρ = 1.06 kg/m^3, Specific heat, C_p=1.008 kJ/kg-K
Thermal conductivity of air, k=0.028 W/m-deg
Kinematic viscosity of air, $\nu = 18.97 \times 10^{-6}\ m^2/s$.
Mean film temperature, T_f = (125+30)/2=77.5°C,
Area of plate surface, A = 0.25 X0.25=0.0625 m^2

To determine: i) Heat loss from plate surfaces for vertical plate

Since there are two surfaces from which heat will be lost to air, therefore,

$$\mathbf{Q = 2 \times h \times A \times (T_s - T_a)}$$

Where, h is convective heat transfer coefficient and is given as

$$Nu = \frac{h \times D}{k} \;\; or \;\; h = \frac{Nu \times k}{D}$$

Nu is Nusselt number for vertical surface is given as

$$Nu = 0.125 \times (Gr \times Pr)^{0.33}$$

Gr is Grashoff number and is expressed as

$$Gr = \frac{L^3 \rho^2 \beta g \Delta T}{\mu^2} = \frac{L^3 \beta g \Delta T}{\nu^2}$$

$$\beta = \frac{1}{T_f} = \frac{1}{273+77.5} = 2.85 \times 10^{-3} \text{ per degree kelvin}$$

$$Gr = \frac{0.25^3 \times 2.85 \times 10^{-3} \times 9.81(125-30)}{(18.97 \times 10^{-6})^2} = \mathbf{1.15 \times 10^8}$$

Pr is Prandtl number and is given as

$$Pr = \frac{\mu c_p}{k} = \frac{\rho \nu c_p}{k} = \frac{1.06 \times (18.97 \times 10^{-6}) \times (1.008 \times 1000)}{0.028} = \mathbf{0.724}$$

$$Gr \times pr = (1.15 \times 10^8) \times 0.724 = \mathbf{0.833 \times 10^8}$$

$$Therefore, \mathrm{Nu} = 0.125 \times (0.83 \times 10^8)^{0.33} = \mathbf{51.36}$$

$$h = Nu \times \frac{k}{l} = 51.36 \times \frac{0.028}{0.25} = \mathbf{5.75\ W/m^2 - K}$$

Therefore, heat loss from both surfaces, $Q = 2 \times 5.75 \times 0.0625 \times (125-30) = \mathbf{68.31\ W}$

ii) When the plate is positioned horizontally

Heat will be lost from upper as well as lower surface of the plate when it is horizontal

a) Heat lost from upper surface:

$$Nu = 0.72(0.83 \times 10^8))^{0.25} = \mathbf{68.72}$$

$$h = Nu\frac{k}{l} = 68.72 \times \frac{0.028}{0.25} = \mathbf{7.69\ W/m^2 - K}$$

$$Q_u = h\, A\, \Delta t = 7.69 \times 0.0625 \times (125-30) = \mathbf{45.69\ W}$$

b) Heat lost from lower surface:

$$Nu = 0.35(0.83 \times 10^8)^{0.25} = \mathbf{33.40}$$

$$h = Nu\frac{k}{l} = 33.40 \times \frac{0.028}{0.25} = \mathbf{3.74\ W/m^2\ K}$$

$$Q_L = h\ A\ \Delta t = 3.74 \times 0.0625 \times (125 - 30) = \mathbf{22.21\ W}$$

$$\therefore \quad Q = Q_u + Q_L = 45.69 + 22.21 = \mathbf{67.90\ W}$$

Ex 17.4 **Determine the boundary layer thickness as a distance of 10 cm and 20 cm from the leading edge of a flat plate when air at a temperature of 45°C flows over it at a speed 1.75 m/s. Kinematic viscosity of air is 17.25 X 10^{-6} m^2/sec and assume parabolic velocity distribution.**

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Solution: Given: Temperatue of air, T_a=45 °C, Kinematic viscosity, υ= 17.25 X10^{-6} m^2/sec
Velocity of air, U_∞ = 0.75 m/sec

To determine: i) Boundary layer thickness

a) At x= .010 m

b) **At x = 0.20 m**

For given velocity distribution, boundary layer thickness is given as

$$\delta = \frac{4.64}{\sqrt{Re_x}}$$

The flow Reynolds number is, $Re_x = (x\ U_\infty)/\vartheta$

$$\text{At } x = 0.10\text{ m} \quad ; \quad Re_x = \frac{0.10 \times 1.75}{17.25 \times 10^{-6}} = \mathbf{10145}$$

$$\text{At } x = 0.20\text{ m} \quad ; \quad Re_x = \frac{0.20 \times 1.75}{17.25 \times 10^{-6}} = \mathbf{20290}$$

$$\therefore \text{At } x = 0.10\text{ m} \ ; \quad \delta_1 = \frac{4.64 \times 0.10}{\sqrt{10145}} = \mathbf{4.6 \times 10^{-3}\ m}$$

$$\therefore \text{At } x = 0.20\text{ m} \ ; \quad \delta_2 = \frac{4.64 \times 0.20}{\sqrt{20290}} = \mathbf{6.51 \times 10^{-3}\ m}$$

Ex 17.5 **An oil at a temperature 60 °C is flowing over a flat plate at a velocity of 0.25 m/sec. The plate is heated to a temperature of 90 °C at a point where boundary layer thickness is 25 mm. Determine the heat transfer per unit area from the plate and heat transfer ceofficient if the temperature profile as measured at this point is represented by**

$$\frac{T-T_s}{T_\infty-T_s} = 1.5\left(\frac{y}{\delta_t}\right) - 0.5\left(\frac{y}{\delta_t}\right)^3$$

Thermal conducivity of oil is 0.871 W/m-K

Solution: Given: Temperatue of plate, T_s=90 °C, Temperature of oil, T_∞= 60 °C
Thermal conductivityof oil, k = 0.871 W/m-K, Velocity of oil = 0.25 m/sec
Boundary layer thickness, δ_t = 25 mm = 0.02 m

To determine: i) Heat transfer per unit area
ii) Convective heat transfer coefficient

i) Heat transfer per unit area is given as

$$\frac{Q}{A} = -k(T_\infty - T_s)\frac{\partial}{\partial y}\left(\frac{T-T_s}{T_\infty - T_s}\right)_{y=0}$$

$$= -k(T_\infty - T_s)\frac{\partial}{\partial y}\left(1.5\left(\frac{y}{\delta_t}\right) - 0.5\left(\frac{y}{\delta_t}\right)^3\right)_{y=0}$$

$$= -k(T_\infty - T_s)\left|\frac{1.5}{\delta_t} - \frac{3\times0.5}{\delta_t}y^2\right|_{y=0} = \frac{1.5k(T_s-T_\infty)}{\delta_t}$$

$$= \frac{1.5\times0.871\times(90-60)}{0.025} = 1568\ \mathrm{W/m^2}$$

(b) Heat transfer coefficient, h

$$h = \frac{Q/A}{(T_s - T_\infty)} = 1568/(90-60) = 52.25\ \mathrm{W/(m^2-K)}$$

Ex 17.6 **A flat plate, 1 mlong and 0.5 m wide is heated electrically so that temperature at its surface is 105 °C. Air flows over this plate with an approach velocity of 2.5 m/sec. If temperature of air is 15 °C, determine**

i. **Total heat transfer rate from the plate to air.**
ii. **Local heat transfer coefficient at a distance of 0.4 m from the leading edge.**

Thermo-physical properties of air at mean temperature are as:

Kinematic viscosiyt, υ= 19.50 X10^{-6} m^2/sec, Prandtl number, Pr = 0.768
Thermal conductivity, k = 0.028 W/m-K ;

Solution: Given: Temperatue of plate, T_s=105 ºC, Temperature of air, T_∞= 15 ºC
Thermal conductivityof air, k = 0.028 W/m-K, Velocity of air, U_∞ = 2.5 m/sec
Kinematic viscosity, υ= 19.50 X10^{-6} m^2/sec, Prandtl number, Pr = 0.768
Length of plate, L = 1 m, Width of plate, W= 0.5 m,
Area of palte, A = 1 X 0.5 = 0.5 m^2

To determine: i) Total heat transfer rate from plate to air
ii) Local heat transfer coefficient at a distance of 0.4 m from the leading edge of the plate

i) Total heat transfer rate from plate to air

First of all Reynolds number for the entire length of the plate is determined by uing the following equation

$$Re_L = \frac{U_\infty L}{\vartheta} = \frac{2.5 \times 1}{19.50 \times 10^{-6}} = 1.282 \times 10^5 < 5 \times 10^5$$

Since Re_L is less than 5 X 10^5, therefore, Nusselt number is expressed as

$$\overline{N}_u = 0.664(R_{el})^{0.5}(Pr)^{0.33}$$

$$= 0.664 \times (1.282 \times 10^5)^{0.5} \times (0.768)^{0.33} = 217.91$$

Therefore, $$\overline{h} = \overline{N}_u \times \frac{k}{L} = 217.91 \times \frac{0.028}{1} = 6.10 \text{ W/(m}^2 - \text{deg)}$$

$$\therefore Q = h \times A \times (T_s - T_\infty) = 6.10 \times 0.5 \times (105 - 15) = 274.50 \text{ W}$$

ii) Local heat transfer coefficient at a distance of 0.4 m from the leading edge

Reynolds number at a distance 0.4 m from leading edge is given as

$$Re_x = \frac{U_\infty x}{V} = \frac{2.5 \times 0.4}{19.50 \times 10^{-6}} = 0.51 \times 10^5$$

Since Re_x is less than 5 X 10^5, therefore flow is laminar and Nusselt number is expressed as

$$N_{ux} = 0.332(Re_x)^{0.5}(Pr)^{0.33} = 0.332 \times (0.51 \times 10^5)^{0.5} \times (0.768)^{0.33} = 68.72$$

$$\therefore h_x = N_{ux} \times \frac{k}{x} = 68.72 \times \frac{0.028}{0.4} = 4.81 \; W/m^2 - K$$