LESSON 21

Unit Solid Angle and Intensity of radiation:

Generally speaking solid angle is that fraction of surface of a sphere that is seen by an observer positioned at the centre of a sphere. The ratio of area of this small surface being observed from centre of the sphere to square of radius of sphere represents solid angle. An observer standing at the centre of a sphere of radius 'R' will see a curve (which is fraction of surface of sphere)

The unit **solid angle** is defined as the angle covered by unit area on a surface of a sphere of unit radius when joined with the centre of the sphere and has been shown in Figure 1. Unit solid angle is measured in the steradians and is expressed as

$$\omega = \frac{A}{R^2}$$

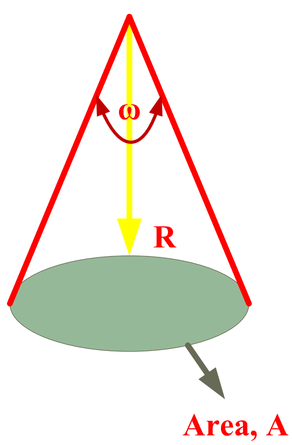

Figure 1

The **intensity of radiation** is defined as the rate of emission of radiation in a given direction from a surface per unit solid angle and per unit projected area of a radiating surface on a plane perpendicular to the direction of radiation.

$$E_b = \pi I_b$$

E_b is the energy emitted and I_b is the intensity of radiations.

Radiation Heat Transfer between Two Black Bodies: Configuration Factor

Radiation heat exchange between two bodies is influenced by the following parameters

i) **Temperature of the individual bodies**

ii) **Radiation properties of the individual bodies such as emissivity**

iii) Orientation of the bodies relative to each other

Orientation of bodies relative to each other means how well one body is able to see the other body. The effect of orientation of the bodies on radiation heat transfer is accounted by considering a factor called configuration factor which is also called shape or view factor or configuration factor. In order to understand the significance of configuration factor let us consider a body 1 which is emitting radiations from its bottom surface only and a part of these radiations is intercepted by each of three bodies 2,3 and 4 as shown in Figure 2.

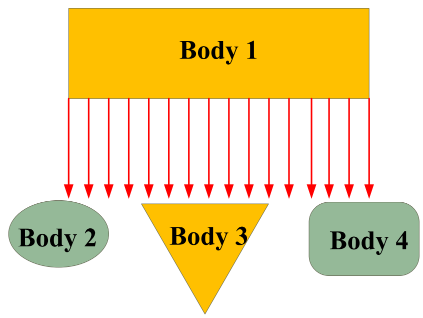

Figure 2

Configuration factor between body 1 and body 2 is denoted by F_{1-2} and is expressed as

$$F_{1-2} = \frac{Radiation\ Energy\ received\ by\ body\ 2\ from\ body\ 1}{Total\ energy\ radiated\ by\ body\ 1}$$

Subscript 1 represents the emitting body and subscript 2 represents the receiving or intercepting body.

Similarly, configuration or view or shape factor between body 2 and body 1 can be expressed as

$$F_{2-1} = \frac{Radiation\ Energy\ received\ by\ body\ 1\ from\ body\ 2}{Total\ energy\ radiated\ by\ body\ 2}$$

Similarly, configuration or view or shape factor between body 1 and body 3 can be expressed as

$$F_{1-3} = \frac{Radiation\ Energy\ received\ by\ body\ 3\ from\ body\ 1}{Total\ energy\ radiated\ by\ body\ 1}$$

Similarly, configuration or view or shape factor between body 1 and body 4 can be expressed as

$$F_{1-4} = \frac{Radiation\ Energy\ received\ by\ body\ 4\ from\ body\ 1}{Total\ energy\ radiated\ by\ body\ 1}$$

Configuration or view or shape factor depends upon the geometry / shape of the bodies involved in the radiation heat exchange and is independent of surface properties and temperatures of the bodies.

A mathematical expression for configuration or view or shape factor can be obtained by considering two black bodies 1 and 2 exchanging heat by radiations when maintained at temperatures T_1 and T_2 respectively. These two bodies are at a distance ‘S’ from each other and having areas A_1 and A_2 respectively as shown in Figure 3.

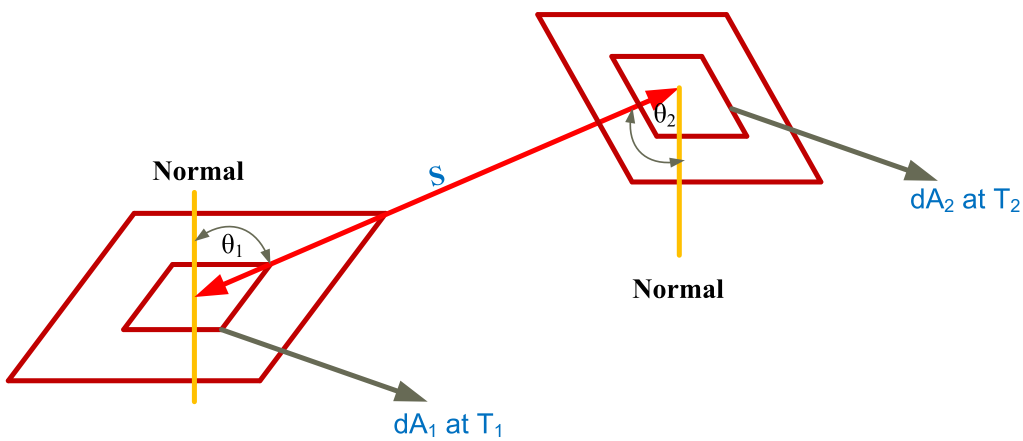

Figure 3

Let us consider two small elements of areas dA_1 and dA_2 at the centers of black bodies 1 and 2 respectively which are exchanging heat with each other by radiation.Heat radiated from element of area dA_1 towards element of area dA_2 is expressed as

$dQ_{(1-2)}$ = Intensity of radiation of element of area dA_1 X projected area of element of area dA_1 along OP X solid angle made by element of area dA_2

$$= I_{b1}\,(dA_1 \cos\theta_1)\left(\frac{dA_2 \cos\theta_2}{S^2}\right) = I_{b1}\,(dA_1)\,(dA_2)\left(\frac{\cos\theta_1 \cos\theta_2}{S^2}\right)$$

This is the total heat lost by the element of area dA_1 and received by the element of area dA_2. Similarly heat is radiated by the element of area dA_2 which is at lower temperature ($T_2 < T_1$) and received by the element of area dA_1 is expressed as

$$dQ_{(2\text{-}1)} = I_{b2}\,(dA_1)\,(dA_2)\left(\frac{\cos\theta_1 \cos\theta_2}{S^2}\right)$$

:. Net heat radiated by the element of area dA_1 towards the element of area dA_2

$$dQ = dQ_{(1\text{-}2)} - dQ_{(2\text{-}1)}$$

$$= (dA_1)\,(dA_2)\left(\frac{\cos\theta_1 \cos\theta_2}{S^2}\right)\left[I_{b1} - I_{b2}\right] \qquad \ldots\ldots\ldots.(1)$$

Since intensity of radiation is given by

$I_b = E_b/\pi = (\sigma T^4)/\pi$

Therefore, $I_{b1} = E_{b1}/\pi = (\sigma T_1^4)/\pi$ and $I_{b2} = E_{b2}/\pi = (\sigma T_2^4)/\pi$

Substituting the values of I_{b1} and I_{b2} in equation (1)

$$dQ = \frac{\sigma}{\pi}(T_1^4 - T_2^4)(dA_1)\,(dA_2)\left(\frac{\cos\theta_1 \cos\theta_2}{S^2}\right) \qquad (2)$$

In order to determine the total energy radiated from body 1 towards body 2, equation (2) is integrated over areas A_1 and A_2.

$$\therefore\; Q_{(1\text{-}2)} = \frac{\sigma}{\pi}(T_1^4 - T_2^4)\int_{A_1}\int_{A_2}\left(\frac{\cos\theta_1 \cos\theta_2 (dA_1)\,(dA_2)}{S^2}\right) \qquad (3)$$

where $Q_{(1\text{-}2)}$ is total heat radiated from body 1 towards body 2

or Net $Q_{(1\text{-}2)} = F_{(1\text{-}2)} A_1 \sigma\,(T_1^4 - T_2^4)$ (4)

Comparing equations (3) and (4), we get

$$\therefore\; F_{(1\text{-}2)} = \int_{A_1}\int_{A_2}\left(\frac{\cos\theta_1 \cos\theta_2}{A_1\,\pi\,S^2}\right)(dA_1)\,(dA_2) \qquad (5)$$

where $F_{(1\text{-}2)}$ is called the configuration factor or shape factor or view factor between the two radiating bodies and is a function of geometry only.

The subscripts 1 and 2 signify that the configuration factor is from body 1 to body 2. The configuration factor from body 2 to body 1 is given by

$$\therefore \quad F_{(2\text{-}1)} = \int_{A_1}\int_{A_2} \left(\frac{\cos\theta_1 \cos\theta_2}{A_2\ \pi\ S^2} \right) (dA_1)\,(dA_2) \qquad (6)$$

The net energy lost by body 1 must be equal to net energy gained by body 2.

$\therefore \quad Q_{(1\text{-}2)} = -\,Q_{(2\text{-}1)} \qquad$ (As $T_2 < T_1$, so -ve sign)

$\therefore \quad F_{(1\text{-}2)}\,A_1\sigma\,(T_1^4 - T_2^4) = -\,F_{(2\text{-}1)}\,A_2\sigma\,(T_2^4 - T_1^4)$

$$\therefore \quad F_{(1\text{-}2)}\,A_1 = F_{(2\text{-}1)}\,A_2 \qquad (7)$$

Equation (9) is known as reciprocal relation between the shape factors.

If Area A_1 is small compared with A_2, then

$$F_{(1\text{-}2)} = \int \frac{\cos\theta_1 \cos\theta_2}{\pi\ S^2}\ (dA2) \qquad (8)$$

REVIEW QUESTIONS:

Q.1 A unit solid angle is equal to

- **a) Ratio of Area and square of radius**
- b) Product of area and radius
- c) Ratio of radius and area
- d) None of the above

Q.2 Heat transfer by radiation between two black bodies is a function of

- a) Temperature of indivisual bodies
- b) Orientation of bodies with respect to each other
- c) Emissivity of indivisual bodies
- **d) All above**

Q.3 Shape / View/ Configuration factor, $F_{1\text{-}2}$ between body 1 and body 2 is equal to

- a) Ratio of energy received by body 1 from body 2 to the energy eitted by body 2
- **b) Ratio of energy received by body 2 from body 1 to the energy eitted by body 1**
- c) Ratio of energy emitted by body 1 to its emissive power
- d) None of the above

Q.4 Configuration factor depends upon

- **a) Shape or geometory of the bodies exchanging heat**
- b) Surface properties of the bodies exchanging heat
- c) Temperature of the bodies exchaning heat
- d) All the above