

LESSON-22

Properties of Configuration Factor:

Properties of shape factor are as:

1. The shape factor is purely a function of geometrical parameters.
2. When two bodies radiating energy with each other only, the shape factor relation is expressed as

$$A_1 F_{1-2} = A_2 F_{2-1} \quad (1)$$
3. The shape factor of convex surface or flat surface with the other surface enclosing the first is always unity. This is because all the radiation coming out from the convex surface is intercepted by the enclosing surface but not vice versa.
4. A concave surface has a shape factor with itself because the radiation energy coming out from one part of the surface is intercepted by another part of the same surface. The shape factor of a surface with respect to itself is denoted by F_{1-1} .

$$F_{1-1} = 0 \quad \text{for convex and flat surface.}$$

If a surface of area A_1 is completely enclosed by a second surface of area A_2 and if A_1 does not see itself ($F_{1-1} = 0$) then $F_{1-2} = 1$

Then substituting the value of $F_{1-2} = 1$ in equation (1),

$$F_{2-1} = \frac{A_1}{A_2}$$

Radiant exchange between coaxial cylinders, bodies placed in enclosures (sphere is kept in a room or box) are examples of this situation.

5. If n surfaces are taking part in radiation heat transfer then the energy radiated by one is always intercepted by the remaining $(n-1)$ surfaces and by the surface itself also

$$\begin{aligned}
 F_{1-1} + F_{1-2} + F_{1-3} \dots\dots\dots F_{1-n} &= 1 \\
 F_{2-1} + F_{2-2} + F_{2-3} \dots\dots\dots F_{2-n} &= 1 \\
 : & \quad : \quad : \\
 : & \quad : \quad : \\
 F_{n-1} + F_{n-2} + F_{n-3} \dots\dots\dots F_{n-n} &= 1
 \end{aligned} \quad (2)$$

In addition to the above equations, the reciprocal relation between any two surfaces also holds good

$$F_{(1-2)} A_1 = F_{(2-1)} A_2 \quad \text{or} \quad F_{(1-3)} A_1 = F_{(3-1)} A_3 \quad \text{and so on.}$$

6. The shape factor between the surfaces A_1 and A_2 is equal to the sum of the shape factors between the surface A_2 and the surfaces which make the surface A_1 , This point is illustrated as shown in Figure 1.

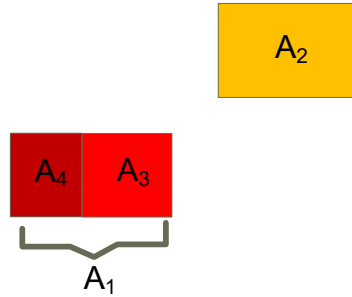


Figure 1

It means that the amount of radiated energy by A_2 and intercepted by A_1 is equal to the sum of the radiated energy intercepted by the areas A_3 and A_4 as shown in Figure 1.

Consider two surfaces of areas A_1 and A_2 are radiating heat to each other as shown in Figure1. Let A_1 be subdivided into A_3 and A_4 (i.e. $A_1 = A_3 + A_4$).

Then the radiant heat exchange between A_1 and A_2 is expressible as

$$Q_{1-2} = Q_{3-2} + Q_{4-2}$$

Considering the surfaces to be black

$$F_{(1-2)} A_1 \sigma (T_1^4 - T_2^4) = F_{(3-2)} A_3 \sigma (T_3^4 - T_2^4) + F_{(4-2)} A_4 \sigma (T_4^4 - T_2^4)$$

As $T_3 = T_4 = T_1$ and A_3 and A_4 are merely sub-divisions of A_1

$$\text{Therefore } F_{(1-2)} A_1 = F_{(3-2)} A_3 + F_{(4-2)} A_4 \quad (3)$$

The above expression shows that

$$F_{1-2} = (F_{3-2} + F_{4-2})$$

For radiant exchange from A_2 to A_1 (divided into A_3 and A_4) one has

$$F_{(2-1)} A_2 = F_{(2-3)} A_3 + F_{(2-4)} A_4$$

$$\text{Therefore } F_{(2-1)} = F_{(2-3)} + F_{(2-4)} \quad (4)$$

7. If the interior surface of a completely enclosed space such as room is subdivided into n parts, each part having a finite area $A_1, A_2, A_3, A_4, \dots, A_n$, then

$$F_{1-1} + F_{1-2} + F_{1-3} \dots\dots\dots F_{1-n} = 1$$

$$F_{2-1} + F_{2-2} + F_{2-3} \dots\dots\dots F_{2-n} = 1$$

$$\vdots \quad \quad \quad \vdots$$

$$\vdots \quad \quad \quad \vdots$$

$$F_{n-1} + F_{n-2} + F_{n-3} \dots\dots\dots F_{n-n} = 1$$

$$\sum_{j=1}^{j=n} F_{i-j} = 1 \quad \text{where } i = 1, 2, 3, \dots, n. \tag{5}$$

The above representation admits the shape factors $F_{1-1}, F_{2-2}, F_{3-3}, \dots, F_{n-n}$, since some of the surface may see themselves if they are concave.

Shape Factor of a Cavity with itself:

Figure 2 shows an irregular cavity having an inner area A_1 and is covered by a flat surface of area A_2 . Configuration factor equations for this arrangement is written as

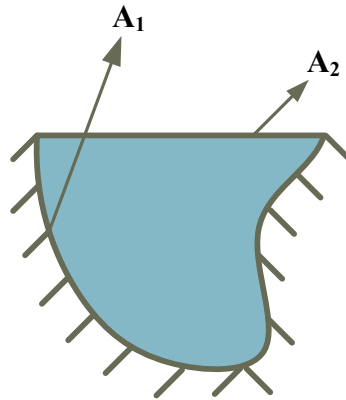


Figure 2

$$F_{1-1} + F_{1-2} = 1 \tag{6}$$

$$F_{2-2} + F_{2-1} = 1 \tag{7}$$

Since the cavity is covered by a flat surface, so $F_{2-2} = 0$,

Substituting the value of $F_{2-2} = 0$ in equation (7), we get

$$F_{2-1} = 1 \tag{8}$$

The reciprocal relation between two surfaces is expressed as

$$F_{1-2} A_1 = F_{(2-1)} A_2$$

Using equation (8), the reciprocal relation can be written as

$$F_{1-2} = (A_2 / A_1) \quad (9)$$

Substituting the values of F_{1-2} in equation (6),

$$F_{1-1} + (A_2 / A_1) = 1$$

$$F_{1-1} = 1 - \frac{A_2}{A_1} \quad (10)$$

Equation (10) represents configuration factor of a cavity with itself.

$$\text{Therefore } F_{1-1} = 1 - F_{1-2} = 1 - \frac{A_2}{A_1} F_{2-1}$$

Substituting the value of F_{2-1} from Equation (7) in above Equation, we get

$$F_{1-1} = 1 - \frac{A_2}{A_1} (1 - F_{2-2}) = 1 - \frac{A_2}{A_1} \quad \text{as } F_{2-2} = 0 \quad (11)$$

The above expression is valid for all types of the cavities as shown in Figures 3 (a), (b), and (c).

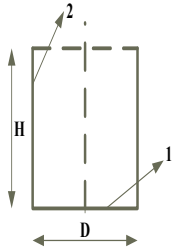


Figure 3 (a)

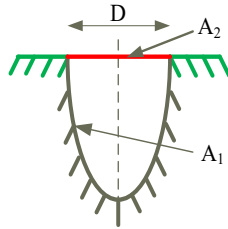


Figure 3 (b)

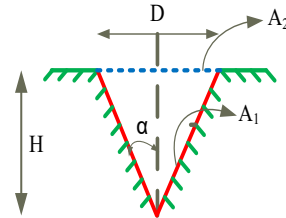


Figure 3 (c)

$$\tan \alpha = \frac{D}{2H}$$

(a) Shape factor of a cylindrical cavity of diameter D and height H with itself

$$F_{1-1} = 1 - \frac{A_2}{A_1} \quad (12)$$

From Figure 3 (a), it can be written that

$$A_2 = \frac{\pi D^2}{4}$$

$$A_1 = \pi DH + \frac{\pi D^2}{4}$$

Substituting the values of A_1 and A_2 in equation (12)

$$F_{1-1} = 1 - \frac{\frac{\pi D^2}{4}}{\pi DH + \frac{\pi D^2}{4}} \quad (13)$$

(b) Shape factor of a hemi-spherical cavity of diameter D with itself

From Figure (b), it can be written that

$$A_2 = \frac{\pi D^2}{4}$$

$$A_1 = \frac{\pi D^2}{2}$$

Substituting the values of A_1 and A_2 in equation (12)

$$F_{1-1} = 1 - \frac{\frac{\pi D^2}{4}}{\frac{\pi D^2}{2}} = 1 - \frac{1}{2} = 0.5 \quad (14)$$

(c) Shape factor of a conical cavity of diameter D and height H with itself

From Figure 3(c), it can be written that

$$A_2 = \frac{\pi D^2}{4}, \quad A_1 = \frac{\pi DL}{2} \quad \text{where } L \text{ is the slant height of the conical cavity}$$

Substituting the values of A_1 and A_2 in equation (12)

$$\begin{aligned}
 F_{1-1} &= 1 - \frac{\frac{\pi D^2}{4}}{\frac{\pi DL}{2}} = 1 - \frac{D}{2L} \\
 &= 1 - 2 \sin \alpha \quad \text{where } \alpha \text{ is half vertex angle} \\
 &= 1 - \frac{D}{2\sqrt{H^2 + \frac{D^2}{4}}} \\
 &= 1 - \frac{D}{\sqrt{4H^2 + D^2}} \tag{15}
 \end{aligned}$$

Shape Factors for Two Perpendicular Plates:

Determination of configuration factor for commonly used geometries such as parallel and perpendicular walls, parallel disks is cumbersome and complicated, therefore in such cases configuration factor is determined with the help of graphs.

Shape factors for a system of Two Perpendicular Plates

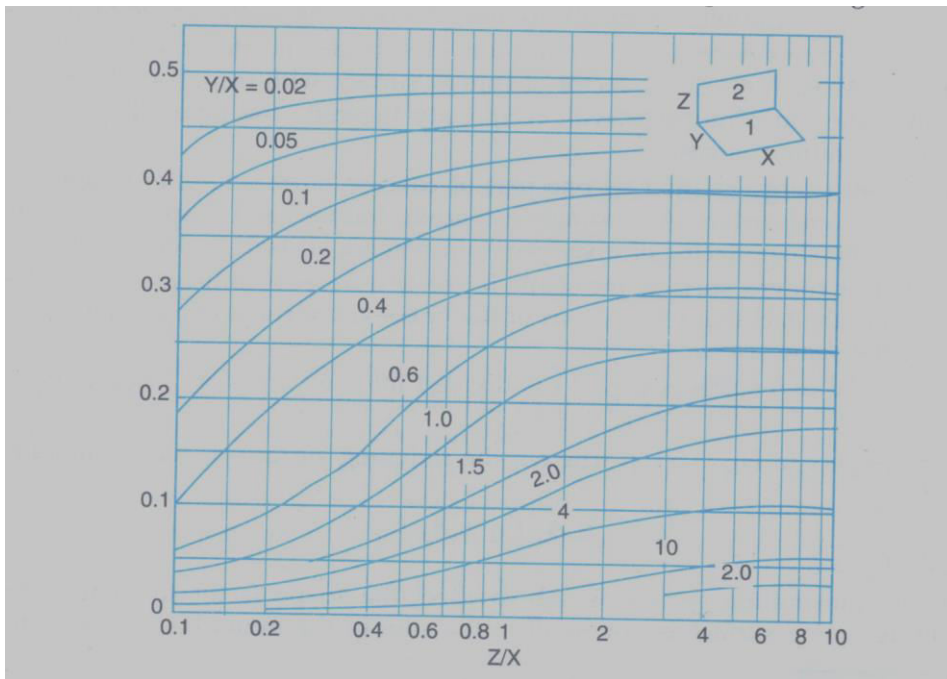


Figure 4 Configuration Factor for two perpendicular plates

Complex Configurations Derivable From Perpendicular Rectangles with Common Edge:

- i) One rectangle is displaced from the common intersection line

$$A_1 F_{1-4} = A_1 F_{1-3} + A_1 F_{1-2}$$

$$\text{Therefore } F_{1-2} = F_{1-4} - F_{1-3} \quad (16)$$

The shape factors F_{1-3} and F_{1-4} may be found easily from the graph as shown in Figure 4.

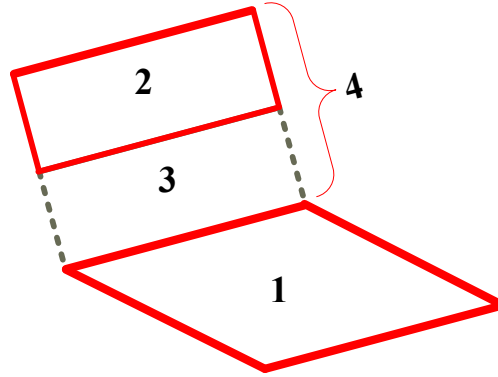


Figure 5

ii) Both rectangles are displaced from the common intersection line.

$$A_1 F_{1-2} = A_5 F_{5-2} - A_3 F_{3-2}$$

$$= (A_5 F_{5-6} - A_5 F_{5-4}) - (A_3 F_{3-6} - A_3 F_{3-4})$$

$$= (A_5 F_{5-6} - A_5 F_{5-4} - A_3 F_{3-6} + A_3 F_{3-4})$$

$$= (A_5 F_{5-6} + A_3 F_{3-4}) - (A_5 F_{5-4} + A_3 F_{3-6})$$

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The shape factors F_{5-6} , F_{3-4} , F_{5-4} and F_{3-6} may be found from graph as shown in Figure 4.

$$F_{1-2} = \frac{1}{A_1} [(A_5 F_{5-6} + A_3 F_{3-4}) - (A_5 F_{5-4} + A_3 F_{3-6})]$$

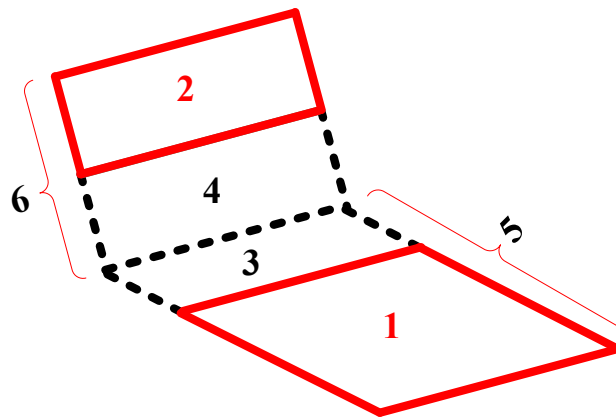


Figure 6

iii) The corner of both rectangles are touching at a point which lies on common intersection line.

$$A_1 F_{1-2} = A_6 F_{6-5} - A_1 F_{1-4} - A_3 F_{3-2} - A_3 F_{3-4} \quad (18)$$

The following reciprocal relation is valid for the above configuration.

$$A_1 F_{1-2} = A_3 F_{3-4}$$

Substituting this in the above equation (18)

$$F_{1-2} = \frac{1}{2A_1} [A_6 F_{6-5} - A_1 F_{1-4} - A_3 F_{3-2}] \quad (19)$$

The shape factors F_{6-5} , F_{1-4} , and F_{3-2} may be found from graph as shown in Figure 4.

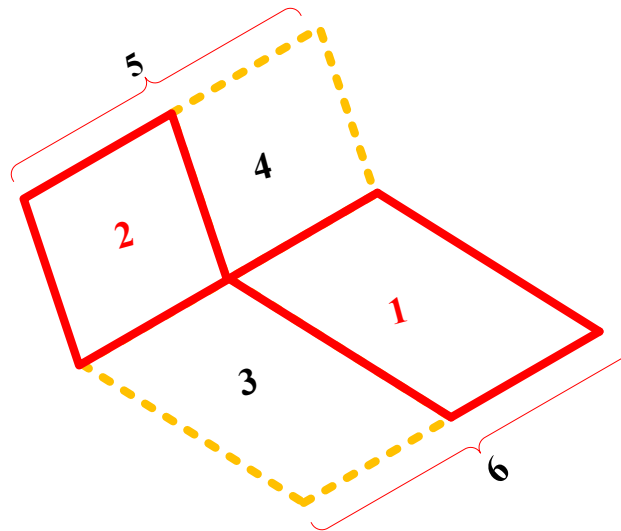


Figure 5

REVIEW QUESTIONS:

Q.1 Configuration factor of a convex surface with itself is always

- a) **Equal to zero**
- b) Less than one
- c) Equal to one
- d) Greater than one

Q.2 Configuration factor of a concave surface with itself

- a) Is equal to zero
- b) Is equal to one
- c) **Does exists**
- d) None of the above

Q.3 Reciprocal theorem is applicable to the two bodies when they are exchanging heat

- a) Are also exchanging heat with a third body
- b) **With each other only**
- c) Both a and b
- d) None of the above

Q.4 Configuration factor is a function of

- a) **Geometrical parameters of bodies**
- b) Temperature of the bodies
- c) Both a and b
- d) None of the above