# LESSOIN-23

**Radiation between Two Infinite Parallel Plates and Proof of Kirchhoff's law of Radiation:**

At a given temperature, the total emissive power of a body is equal to its absorptivity multiplied by total emissive power of a perfect black body at that temperature.

Therefore $E = \alpha E_b$

But the ratio of total emissive power of a body to the total emissive power of a black body at the same temperature is called the emissivity of the body and is numerically equal to absorptivity.

$$\therefore \quad \alpha = \varepsilon = \frac{E}{E_b}$$

Consider two bodies C and D whose absorptivity are $\alpha_c$ and $\alpha_d$ as shown in Figure 1.

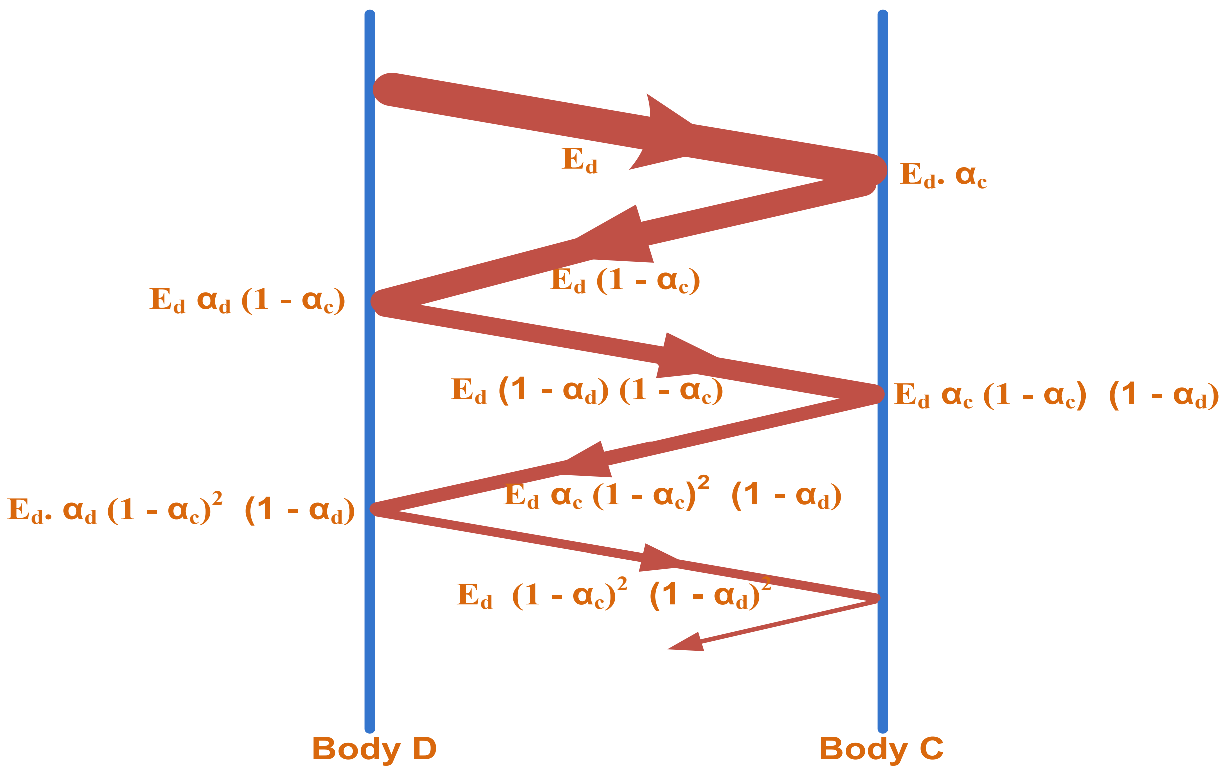


Figure 1

Considering the energy emitted by the body D.

(1) D emits the energy = $E_d$ (1)

(2) C absorbs energy = $E_d . \alpha_c$ (2)

and reflects energy=$E_d (1 - \alpha_c)$ (3)

(3) D absorbs energy = $E_d \alpha_d (1 - \alpha_c)$ (4)

and reflects energy = $E_d (1 - \alpha_d) (1 - \alpha_c)$ (5)

(4) C absorbs energy = $E_d \alpha_c (1 - \alpha_c) (1 - \alpha_d)$ (6)

and reflects energy = $E_d (1 - \alpha_c)^2 (1 - \alpha_d)$ (7)

(5) D absorbs energy = $E_d . \alpha_d (1 - \alpha_c)^2 (1 - \alpha_d)$ (8)

and reflects energy = $E_d (1 - \alpha_c)^2 (1 - \alpha_d)^2$ (9)

and so on upto $\infty$ times.

Considering the energy emitted by the body C.

(1) C emits the energy = $E_c$ (10)

(2) D absorbs energy = $E_c . \alpha_d$ (11)

and reflects energy=$E_c (1 - \alpha_d)$ (12)

(3) C absorbs energy = $E_c \alpha_c (1 - \alpha_d)$ (13)

and reflects energy = $E_c (1 - \alpha_c) (1 - \alpha_d)$ (14)

(4) D absorbs energy = $E_c \alpha_d (1 - \alpha_d) (1 - \alpha_c)$ (15)

and reflects energy = $E_c (1 - \alpha_d)^2 (1 - \alpha_c)$ (16)

(5) C absorbs energy = $E_c . \alpha_c (1 - \alpha_d)^2 (1 - \alpha_c)$ (17)

and reflects energy = $E_c (1 - \alpha_d)^2 (1 - \alpha_c)^2$ (18)

and so on upto infinite number of times.

Considering equations (1), (4), (8), (11), (I5), net energy lost by the body D is expressed as

= Energy emitted by it – energy absorbed by it

$\therefore q_{(dc)} = E_d - [E_d \alpha_d (1 - \alpha_c) + E_d . \alpha_d (1 - \alpha_c)^2 (1 - \alpha_d) + \ldots] - [E_c . \alpha_d + E_c \alpha_d (1 - \alpha_d) (1 - \alpha_c) + \ldots]$

Assuming $(1 - \alpha_c)(1 - \alpha_d) = K$

$\therefore q_{(dc)} = E_d - E_d \alpha_d (1 - \alpha_c) [1 + K + K^2 + \ldots] - E_c . \alpha_d [1 + K + K^2 + \ldots]$

But $1 + K + K^2 + \ldots = (1 - K)^{-1}$

$\therefore q_{(dc)} = E_d - (1 - K)^{-1} [E_d \alpha_d (1 - \alpha_c) + E_c . \alpha_d]$

$$= E_d - \frac{[\alpha_d (1-\alpha_c) E_d + E_c \alpha_d]}{(1-K)} = \frac{E_d (1-K) - [\alpha_d (1-\alpha_c) E_d + E_c \alpha_d]}{(1-K)}$$

Substituting the values of K

$$q_{(dc)} = \frac{E_d [\alpha_d + \alpha_c - \alpha_d \alpha_c] - [\alpha_d (1-\alpha_c) E_d + E_c \alpha_d]}{[1 - (1-\alpha_d)(1 - \alpha_c)]} = \frac{E_d \alpha_c - E_c \alpha_d}{[\alpha_d + \alpha_c - \alpha_c \alpha_d]} \quad (19)$$

**i) If originally both bodies are at same temperature**

Then $q_{(dc)} = 0$ , Then equation (19) can be written as

$$\therefore E_d \alpha_c = E_c \alpha_d$$

Assuming C as black body

Then $\alpha_c = 1$

$$\alpha_d = \frac{E_d}{E_c}$$

as $E_c = E_b$ $\quad \therefore \alpha_d = \frac{E_d}{E_b}$

Subscription 'b' represents black body.

However, $E_d / E_b$ is the emissivity of body 'D' according to definition of emissivity.

Therefore, $\varepsilon_d = \alpha_c$ This is the statement of Kirchoff's law and hence it is proved.

**ii) If both the bodies are at different temperatures**

Using equation (19)

$$q_{dc} = \frac{E_d \alpha_c - E_c \alpha_d}{[\alpha_d + \alpha_c - \alpha_c \alpha_d]}$$

**According to Stefan-Boltzman Law**

$$E_d = \varepsilon_d \sigma_b T_1^4$$

$$E_c = \varepsilon_c \sigma_b T_2^4$$

Substituting these values in equation (19), we get

$$q_{dc} = \frac{\varepsilon_d \varepsilon_c \sigma_b T_1^4 - \varepsilon_d \varepsilon_c \sigma_b T_2^4}{[\varepsilon_d + \varepsilon_c - \varepsilon_c \varepsilon_d]}$$

$$q_{dc} = \frac{\sigma_b\left(T_1^4 - T_2^4\right)}{\left[\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_d} - 1\right]} \tag{20}$$

## Radiation Shields

Generally, the shields are used for reducing the heat radiation from one plate to another plate. A shield 3 is placed in between the two plates as shown in Figure 2 and plates and radiation shield are at temperatures $T_1$, $T_2$ and $T_3$.

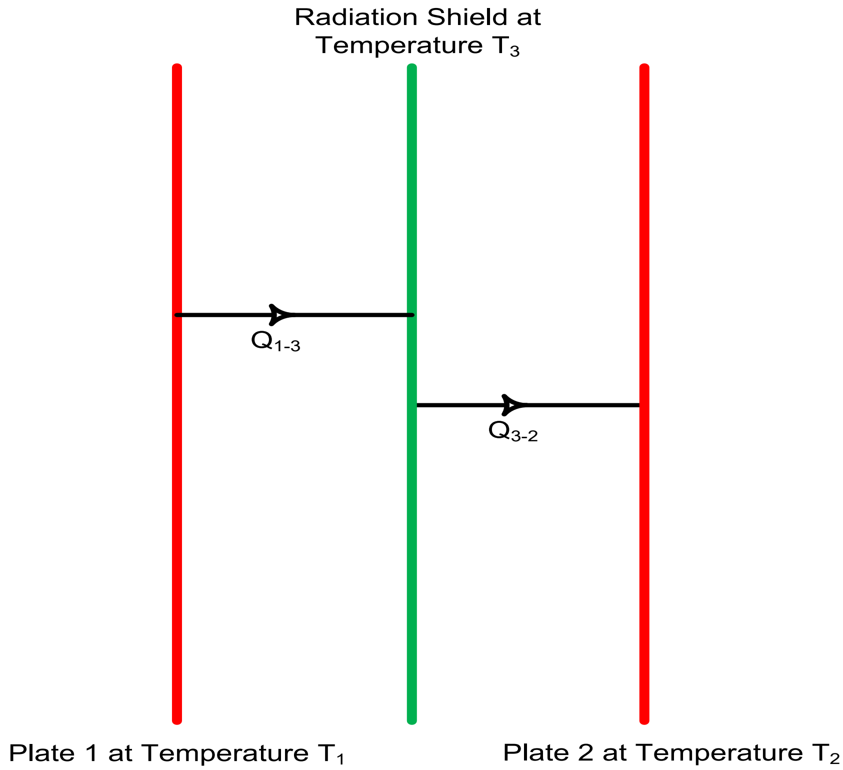


Figure 2

Assuming there is no temperature drop in the shield and considering the system is in steady state condition, we can write down the heat flow equation as

$$Q_{1-3} = \frac{A\sigma(T_1^4 - T_3^4)}{\left[\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right]} \tag{21}$$

$F_{1-3}$= 1 as plates are parallel to each other

Similarly,

$$Q_{3-2} = \frac{A\sigma(T_3^4 - T_2^4)}{\left[ \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1 \right]} \tag{22}$$

For steady state conditions, $Q_{1-3}=Q_{3-2}$

$$\frac{(T_1^4 - T_2^4)}{\left[ \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1 \right]} = \frac{(T_3^4 - T_2^4)}{\left[ \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1 \right]} \tag{23}$$

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$, equation (23) becomes

$$T_3^4 = \frac{1}{2}\left(T_1^4 + T_2^4\right) \tag{24}$$

Substituting the value of $T_3$ from equation (24) in equation (21), we get

$$Q_{1-3} = \frac{1}{2}\frac{A\sigma(T_1^4 - T_3^4)}{\left[ \frac{2}{\varepsilon} - 1 \right]} \tag{25}$$

If there is no radiation shield present between plates 1 and 2, heat radiated is expressed as

$$Q_{1-2} = \frac{A\sigma(T_1^4 - T_2^4)}{\left[ \frac{2}{\varepsilon} - 1 \right]} \tag{26}$$

Comparing equations (25) and (26), we get

$$Q_{1-2(\text{with shield})} = \frac{1}{2} \times Q_{1-2(\text{without shield})}$$

It means that with the addition of radiation shield, heat transfer rate is reduced to half of that of without the presence of radiation shield between two parallel bodies exchanging heat with each other by radiation. If 'n'shields are present between the two radiating bodies, then the heat transfer will be expressed as

$$Q_{1-2(with\ 'n'\ shields)} = \left(\frac{1}{n+1}\right)\frac{A\sigma(T_1^4 - T_3^4)}{\left[ \frac{2}{\varepsilon} - 1 \right]} \tag{27}$$