LESSON-24

SOLVED EXAMPLES ON RADIATION

Ex 24.1 **A circular glass window of diameter 10 cm has been provided on the wall of a furnace and inside temperature of furnace is 2000 °C. Determine loss of heat by radiation from glass window if transmissivity glass is 0.090 and stefan-Boltzman constant is 5.68 X 10^{-8} W/m²-K⁴.**

Solution: **Given: Diameter of window, d=10 cm= 0.1 m,**
Area of window, A = π/4 X d² = 3.142/4 X $(0.01)^2$ = 7.855 X 10^{-5} m²
Inside Temperature of furnace, T=2000 °C= 2273 K
Transmissivity of glass, τ = 0.090
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m²-K⁴.

To determine: i) Heat loss by radiation from glass window

$$Q = \sigma_b A T^4 \times \tau$$

$$Q = 5.67 \times 10^{-8} \times \mathbf{7.855\ X\ 10^{-5}} \times 2273^4 \times 0.09 = \mathbf{10.27\ W}$$

Ex 24.2 **A mirror of 5 cm diameter receives energy from the Sun at the rate of 5 W and attains a temperature of 310 K. The mirror is suspended in atmospheric air and temperature of air is 300 K. Determine the reflectivity of the mirror if convective heat transfer coefficient on both surfaces of the plate is stated to be 80 W/m²-deg.**

Solution: **Given: Diameter of mirror, d=5 cm= 0.05 m,**
Area of window, A = π/4 X d² = 3.142/4 X $(0.05)^2$ = 1.96 X 10^{-3} m²
Temperature of mirror, T_1=310 K, Temperature of air, T_2=280 K,
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m²-K⁴.
Energy received by mirror, Q= 5 W
Convective heat transfer coefficient, h = 80 W/m²-deg

To determine: i) Reflectivity of mirror

$$\text{Reflectivity}, \rho = \frac{Q_r}{Q}$$

$$\text{Where, } Q_r \text{ is energy reflected}$$

Since transmissivity of mirror is zero, therefore

$$Q_r = \text{Energy Received by mirror} - \text{Energy lost by mirror}$$

Energy lost by convection from both sides of the mirror

$= 2 X h X A X (T_1 - T_2)$

$$= 2 \times 80 \times 1.96 \times 10^{-3} \times (310 - 300) = \mathbf{3.14\ W}$$

$$\text{Therefore}, Q_r = 5 - 3.14 = 1.86\ W$$

$$\therefore \text{Reflectivity}, \rho = \frac{Q_r}{Q} = \frac{1.86}{5} = 0.372$$

Ex 24.3 **Determine the temperature of a black body which is radiating heat to the walls of a room which are maintained at 200 °C. The walls have a surface area of 0.2 m^2 and thermal conductivity of wall material is 0.96 W/m-deg. Thickness of a wall is of 8 cm thickness and outer surface of the wall is at 40 °C. Area of black body is 0.05 m^2. Neglect the loss of heat through the roof of the room.**

Solution: **Given: Area of black body, A_1 = 0.05 m^2**
Temperature of inner surface of wall, T_1=473 K,
Temperature of outer surface of wall, T_2=313 K,
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m^2-K^4.
Thermal conductivity of wall material, k=0.96 W/m-deg
Surface area of wall, A_2=0.2 m^2
Thickness of wall, δ = 8 cm = 0.08 m

To determine: i) Temperature of black body, T_b

From the given conditions, it is clear that heat raddiated by the black body to the room is equal to the heat conducted by inner surfaces to outer surfaces of walls of the room. Therefore,

Heat radiated by black body, Q_r =
Heat conducted through the four walls

$$\sigma_b A(T_b^4 - T_w^4) = 4 \times \frac{kA\,(T_1 - T_2)}{\delta}$$

$$5.67 \times 10^{-8} \times 0.05(T_b^4 - 473^4) = 4 \times \frac{0.96 \times 0.2 \times (473 - 313)}{0.08} = \mathbf{1536\ W}$$

$$T_b^4 = \frac{1536}{5.67 \times 10^{-8} \times 0.05} + 473^4 = \mathbf{5.92 \times 10^{11}}$$

∴ Temperature of the black body, $\mathbf{T_b = 877.1\ K}$

Ex 24.4 **Considering a furnace to be black body which is at 1500 K, determine the following**

(i) Monochromatic radiant flux density at 2 μm wavelength
(ii) Wavelength at which emission is maximum
(iii) Total emissive power

Solution: **Given: Temperature of furnace, T_b=1500 K,**
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m^2-K^4
Wavelength, λ = 2 μm = 2 X 10^{-6} m

To determine: i) Monochromatic radiant flux density at 2 μm wavelength

Monochromatic radiant fux density of a black body is given as

$$\mathbf{(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\,[C_2/\lambda T] - 1}}$$

$$= \frac{0.374 \times 10^{-15} \times (2 \times 10^{-6})^{-5}}{\exp\,[1.4388 \times 10^{-2}/2 \times 10^{-6} \times 1500] - 1}$$

$$= \mathbf{9.73 \times 10^{10}\ W/m^2\ per\ meter\ wavelength}$$

ii) **Wavelength at which emission is maximum**

From Wien's displacement law; $\lambda_{max}\, T = 2.898 \times 10^{-3}$

$$\therefore\ \lambda_{max} = \frac{2.898\times10^{-3}}{1500} = \mathbf{1.932 \times 10^{-6} m}$$

iii) **Total emissive power, E_b**

Total emissive power is given by Stefan – Boltzman law,

$$\mathbf{E = \sigma_b T^4}$$

$$= 5.67 \times 10^{-8} \times (1500)^4 = \mathbf{287.04\ kW/m^2}$$

Ex. 24.5 **Air at 22 °C is surrounding a cylinder having outside diameter of 3 cm and length 1.5 m. Temperature and emissivity of outer surface of cylinder are 110 °C and 0.3 respectively. If convctive heat transfer coefficient is 15 W/m²-deg, determine the heat transfer by radiation and convection from the cylinder to air. Also determine the overall coefficient of heat transfer by combined mode of convection and radiation.**

Solution: **Given: Temperature of air, T_2= 22 °C= 22+273= 295 K**
Temperature of outer surface of cylinder, T_2=110 °C = 383 K,
Outer diameter of cylinder, D = 3 cm=0.03 m,
Lenth of cylinder, L = 1.5 m
Convective heat transfer coefficient, h = 15 W/m²-deg
Emissivity of cylinder, ε = 0.3,
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m²-K⁴.

To determine: i) Heat transfer by radiation
ii) Heat transfer by convection
iii) Overall heat transfer coefficient for combined convection and radiation heat transfer

i) **Heat transfer by radiation**

$$\mathbf{Q_r(radiation) = \in \sigma_b A\, (T_1^4 - T_2^4)}$$

Surface area of pipe, $\mathbf{A = \pi dl} = \pi \times 0.03 \times 1.5 = 0.141\ m^2$

$$Therefore, \quad Q_r = 0.3 \times 5.67 \times 10^{-8} \times 0.141 \times (383^4 - 295^4) = 28.26\ W$$

ii) **Heat transfer by convection**

$$\mathbf{Q_c(convection) = h\, A\, (T_1 - T_2)}$$

$$= 15 \times 0.141 \times (383 - 295) = \mathbf{164.97\ W}$$

iii) **Overall hat transfer coefficient**

The equation governing the total heat transfer by convection and radiation form outer surface of cylinder to air is given as

$$\mathbf{Q_t = U\, A\, (T_1 - T_2)}$$

$$\mathbf{Where, Q_t(total) = Q_r + Q_c} = 28.26 + 164.97 = \mathbf{193.23\ W}$$

U is overall heat transfer coefficient for combined convection and radiation

$$\therefore U = \frac{Q_t}{A\, \Delta T} = \frac{193.23}{0.141(373 - 295)} = \mathbf{17.56\ W/m^2 - deg}$$

Ex 24.6 **For a black body maintained at a temperature of 1200 K and having an area of 1.2 m^2, determine (a) the total rate of energy emission (b) the intensity of normal radiation (c) the intensity of radiation along a direction 45° to the normal, and (d) the wavelength of maximum monochromatic emissive power.**

Solution: **Given: Temperature of black body, T_b= 1200 K**
Area of black body, A = 1.2 m^2
Angle, Φ = 45^o
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m^2-K^4.

To determine: i) Rate of energy emission

Using Stefan-Boltzmann equation,

$$E_b = \sigma_b A T^4 = (5.67 \times 10^{-8}) \times 1.2 \times 1200^4 = \mathbf{141\ kW}$$

ii) Intensity of normal radiation

$$I_n = \frac{\sigma_b T^4}{\pi} = \frac{5.67 \times 10^{-8} \times 1200^4}{\pi} = \mathbf{44.90\ k\ W/m^2}$$

iii) Intensity of radiation along direction 45^o to normal

Using Lambert's cosine law:

$$I_\emptyset = I_n \cos\emptyset = 44900 \times \cos 45° = 31.75\ \mathrm{kW/m^2\ steradian}$$

iv) Wavelength of maximum monochromatic emissive power

Using Wien's displacement law, the wavelength λ_{max} for maximum monochromatic emissive power is:

$$\lambda_{max} = \frac{2.898 \times 10^{-3}}{T} = \frac{2.898 \times 10^{-3}}{1200} = \mathbf{2.415 \times 10^{-6} m}$$

Ex 24.7 **Two infinitely long parallel plates are placed opposite to each other and are maintained at 640 K and 590 K respectively. Determine the net heat flux between these plates if one has an emissivity of 0.8 and other an emissivity of 0.6. How this heat flux will be affected if the plates are assumed to be black?**

Solution: **Given: Temperature of plate 1, T_1= 640 K**
Temperature of plate 2, T_2= 590 K
Emissivity of plate 1, ε_1 =0.8, Emissivity of plate 2, ε_2 =0.8
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m^2-K^4.

To determine: i) Heat flux between the plates

Heat flux is heat transfer per unit area and is given as

$$Q_{12} = (F_g)_{12} A_1 \sigma_b (T_1^4 - T_2^4)$$

For infinite long parallel planes which see each other and nothing else,

$$A_1 = A_2$$

$$\therefore\ (F_g)_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{1}{\frac{1}{0.8} + \frac{1}{0.6} - 1} = \mathbf{0.52}$$

Therefore, $Q_{12} = 0.52 \times 1 \times 5.67 \times 10^{-8}(640^4 - 590^4) = \mathbf{1373.91\ W/m^2}$

ii) If both the plates are assumed to be black,

$$\mathbf{Q_{12} = A_1\sigma_b(T_1^2 - T_2^4)}$$
$$= 1 \times (5.67 \times 10^{-8}) \times (640^4 - 590^{\circ}) = \mathbf{2642.3\ W/m^2}$$

Ex 24.8 **Two infinitely long parallel square plates each of 2 m² area are separated by a thin metalic sheet having surface emissivity of 0.1. Plate 1 is at temperature 600 K and its surface emissivity is 0.5 while the second plate is at a temperature of 400 K and surface emissivity is 0.8. Determine the temperature of thin metallic sheet and heat transfer between the parallel plates. Also determine the heat transfer rate between the plates in the absence of metallic sheet.**

Solution: **Given: Temperature of plate 1, T_1= 600 K**
Temperature of plate 2, T_2= 400 K, Area of each plate = 2 m²
Emissivity of plate 1, ε_1 =0.5, Emissivity of plate 2, ε_2 =0.9
Emissivity of metallic sheet, ε_3 = 0.1
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m²-K⁴.

To determine: i) Temperature of metallic sheet
ii) Heat transfer between plate 1 and plate 2 in absence of metallic sheet

Let suffix 3 designate the sheet which has been inserted between the two plates.

Heat flow from plate 1 to metallic sheet,

$$\mathbf{Q_{13} = (F_g)_{13}\, A_1\sigma_b(T_1^4 - T_3^4)}$$

Now $$(F_g)_{13} = \frac{1}{1/\epsilon_1 + 1/\epsilon_3 - 1} = \frac{1}{1/0.5 + 1/0.1 - 1} = \mathbf{0.091}$$

$$\therefore\ Q_{13} = 0.09374\ A_1\sigma_b(600^4 - T_3^4) \qquad \ldots.(1)$$

Heat flow from sheet to plate 2,

$$\mathbf{Q_{32} = (F_g)_{32}\, A_3\sigma_b(T_3^4 - T_2^4)}$$

Now $$(F_g)_{32} = \frac{1}{1/\epsilon_3 + 1/\epsilon_2 - 1} = \frac{1}{1/0.1 + 1/0.9 - 1} = \mathbf{0.0989}$$

$$\therefore\ Q_{32} = 0.0989\ A_3\sigma_b(T_3^4 - 400^4) \qquad \ldots.(2)$$

Under steady state conditions,

$$\mathbf{Q_{13} = Q_{32}}$$

$$\therefore\ \mathbf{0.091\ A_1\sigma_b(600^4 - T_3^4) = 0.0989\ A_3\sigma_b(T_3^4 - 400^4)}$$

Since both the plates and metallic sheet are infinitely long and parallel to each other, therefore, $A_1 = A_3$,

$$\therefore\ \mathbf{0.091\left(600^4 - T_3^4\right) = 0.0989\,(T_3^4 - 400^4)}$$

$$T_3^4 = \frac{0.091 \times 600^4 + 0.0989 \times 400^4}{0.091 + 0.0989} = \mathbf{7.54 \times 10^{10}}$$

$$T_3 = \mathbf{524.07\ K}$$

In order to determine the heat transfer between the plate 1 and plate 2 with metallic sheet present, substitute the value of T_3 in either of the equation 1 or equation 2.

$$= 0.091 \times 2 \times (5.67 \times 10^{-8}) \times (600^4 - 524.07^4) = \mathbf{558.88\ W}$$

ii) Heat transfer between plate 1 and plate 2 in absence of metallic sheet

The rate of heat interchange between the two plates is given by:

$$Q_{12} = F_{12} A_1 \sigma_b (T_1^4 - T_2^4)$$

For infinite long parallel plates, $A_1 = A_2$ and shape factor is given as

$$\therefore\ \mathbf{(F)_{12}} = \frac{\mathbf{1}}{\frac{\mathbf{1}}{\boldsymbol{\epsilon_1}} + \frac{\mathbf{1}}{\boldsymbol{\epsilon_2}} - \mathbf{1}} = \frac{1}{\frac{1}{0.5} + \frac{1}{0.9} - 1} = \mathbf{0.47}$$

$$\therefore\ Q_{12} = 0.47 \times 2 \times 5.67 \times 10^{8}(600^4 - 400^4)$$

$$\mathbf{= 5543\ Watts}$$

Presence of metallic sheet decreases the heat transfer between the plates by 5543/558.88 = **9.91 times**.

Ex 24.9 **Two opposite sides of a very long rectangular channel are heated electrically in such a way that temperature of the sides are maintianed at 1000 K and 600 K respectively. Determine the heat exchange per unit area between these sides if emissivity of high and low temperature sides is 0.8 and 0.5 respectively. The heat exchange by radiation between high and low temperature sides is reduced by 30 percent of its origional value by placing a radiation shield between these sides. Determine the emissivity and temperature of the radiation shield.**

Solution: **Given: Temperature of plate 1, T_1= 1000 K**
Temperature of plate 2, T_2= 600 K,
Emissivity of plate 1, ε_1 =0.8, Emissivity of plate 2, ε_2 =0.5
Stefan-Bolzmann constant, σ_b= 5.68 X 10^{-8} W/m^2-K^4

To determine: i) Heat transfer between plate 1 and plate 2 without radiation shield

ii) Emissivity and temperature of radiation shield

i) Heat transfer between plate 1 and plate 2 without radiation shield

$$Q_{12} = \left(F_g\right)_{12} A_1 \sigma_b \left(T_1^4 - T_2^4\right)$$

For infinite long parallel plates, $A_1 = A_2$

$$\left(\mathbf{F_g}\right)_{12} = \frac{\mathbf{1}}{\frac{\mathbf{1}}{\epsilon_1} + \frac{\mathbf{1}}{\epsilon_2} - \mathbf{1}} = \frac{1}{\frac{1}{0.8} + \frac{1}{0.5} - 1} = \frac{1}{1.25 + 2 - 1} = \mathbf{0.44}$$

$$\therefore\ Q_{12} = 0.44 \times 1 \times (5.67 \times 10^{-8})) \times (1000^4 - 600^4) = \mathbf{21.71\ kW/m^2}$$

ii) Emissivity and temperature of radiation shield

When a radiation shield with emissivity ϵ_3 on both sides is placed between the two plates, then shape factor is given as

$$\left(\mathbf{F_g}\right)_{12} = \frac{\mathbf{1}}{\left[\left(\frac{\mathbf{1}}{\epsilon_1} - \mathbf{1}\right) + \frac{\mathbf{1}}{\mathbf{F_{13}}} + \left(\frac{\mathbf{1}}{\epsilon_3} - \mathbf{1}\right)\frac{\mathbf{A_1}}{\mathbf{A_3}}\right] + \left[\left(\frac{\mathbf{1}}{\epsilon_3} - \mathbf{1}\right) + \frac{\mathbf{1}}{\mathbf{F_{32}}} + \left(\frac{\mathbf{1}}{\epsilon_2} - \mathbf{1}\right)\frac{\mathbf{A_3}}{\mathbf{A_2}}\right]}$$

As both the plates and radiation shield are parallel to each other and infinitely long, therefore,

$A_1 = A_2 = A_3$ and $F_{13} = F_{32} = 1$

$$\therefore\ \left(F_g\right)_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2}{\epsilon_3} - 2} = \frac{1}{\frac{1}{0.8} + \frac{1}{0.5} + \frac{2}{\epsilon_3} - 2}$$

Since the radiant heat interchange is limited to 30% of original value by placing a radiation shield between plate 1 and palte 2, therefore

$$\frac{30}{100} \times 21710 = \left(F_g\right)_{12} \times 1 \times (5.67 \times 10^{-8}) \times (1000^4 - 600^4)$$

$$\left(F_g\right)_{12} = \mathbf{0.132}$$

$$0.132 = \frac{1}{\frac{1}{0.8} + \frac{1}{0.5} + \frac{2}{\epsilon_3} - 2}$$

$$\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2}{\epsilon_3} - 2 = 7.57$$

$$\frac{1}{0.8} + \frac{1}{0.5} + \frac{2}{\epsilon_3} - 2 = 7.57$$

$$1.25 + 2 + \frac{2}{\epsilon_3} - 2 = 7.57$$

$$\frac{2}{\epsilon_3} = 7.57 - 1.25 = 6.32$$

$$\epsilon_3 = \frac{2}{6.32} = \mathbf{0.316}$$

Under steady state conditions

$$Q_{12} = Q_{13} = Q_{32}$$

$$\frac{30}{100} \times 21710 = \frac{A_1\, \sigma_b (T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{1 \times 5.67 \times 10^{-8} (1000^4 - T_3^4)}{\frac{1}{0.8} + \frac{1}{0.316} - 1}$$

$$\frac{1 \times 5.67 \times 10^{-8} (1000^4 - T_3^4)}{1.25 + 3.16 - 1} = \frac{1 \times 5.67 \times 10^{-8} (1000^4 - T_3^4)}{3.41}$$

$$(1000^4 - T_3^4) = \frac{30}{100} \times \frac{21710 \times 3.41}{5.67 \times 10^{-8}} = 3.91 \times 10^{11}$$

$$T_3^4 = 1000^4 - 3.91 \times 10^{11} = 6.07 \times 10^{11}$$

$$\therefore T_3 = (2546 \times 10^8)^{1/4} = \mathbf{882.95\ K}$$