LESSON 26

Overall Heat Transfer Coefficient

1. Plane Wall:

Consider a wall of thickness 'δ' made of a material having thermal conductivity 'k' and temperatures of wall surfaces are T_{s1} and T_{s2}. Left side of the wall is in contact with a hot fluid at temperature T_h and the right side is in contact with a cold fluid at temperature T_c as shown in Figure 1. Let h_i and h_o are convective heat transfer coefficients of hot and cold fluids with wall surfaces respectively. Under steady state conditions, heat transferred from hot fluid to wall surface is equal to heat conducted through the wall and it is equal to heat convected to the cold fluid from wall surface.

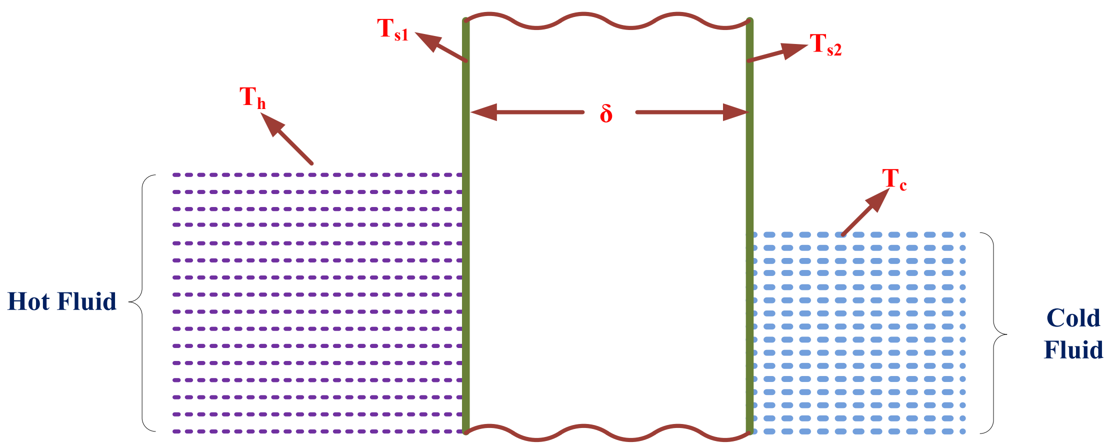

Figure 1

Heat transferred to wall surface by hot fluid is expressed as

$$Q = h_i\, A\, (T_h - T_{s1})$$

$$\frac{Q}{h_i A} = (T_h - T_{s1}) \qquad (1)$$

Heat transferred through the wall by conduction is expressed as

$$Q = k\,A\,\frac{(T_{s1} - T_{s2})}{\delta}$$

$$\frac{Q\delta}{k\,A} = (T_{s1} - T_{s2}) \qquad (2)$$

Heat convected to cold fluid by wall surface is expressed as

$Q = h_o A (T_{s2}-T_c)$

$$\frac{Q}{h_o A} = \left(T_{s2} - T_c\right) \quad (3)$$

Adding both sides of equations (1), (2) and (3), we get

$$Q\left(\frac{1}{h_i A} + \frac{\delta}{kA} + \frac{1}{h_o A}\right) = \left(T_h - T_c\right)$$

$$Q = \frac{\left(T_h - T_c\right)}{\left(\frac{1}{h_i A} + \frac{\delta}{kA} + \frac{1}{h_o A}\right)} \quad (4)$$

We know that heat transfer in a heat exchanger can be expressed as

$$Q = UA\,(T_h - T_c) \quad (5)$$

Comparing equations (4) and (5), we can write

$$UA\left(T_h - T_c\right) = \frac{\left(T_h - T_c\right)}{\left(\frac{1}{h_i A} + \frac{\delta}{kA} + \frac{1}{h_o A}\right)}$$

$$\frac{1}{UA} = \left(\frac{1}{h_i A} + \frac{\delta}{kA} + \frac{1}{h_o A}\right) \quad (6)$$

$$\frac{1}{U} = \left(\frac{1}{h_i} + \frac{\delta}{k} + \frac{1}{h_o}\right) \quad (7)$$

Equation (7) represents overall heat transfer coefficient for a plane wall.

2. **Double Pipe Heat Exchanger**

Consider a double pipe heat exchanger of length ‘L’ in which hot fluid and cold fluid are flowing through inner and outer pipes respectively as shown in Figure 2.

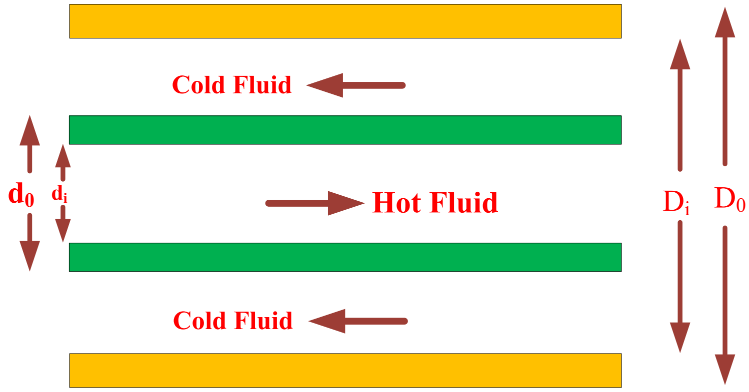

Figure 2

Let

h_i and h_o are convective heat transfer coefficients of hot and cold fluids with pipe surfaces respectively.

d_i and d_o are inside and outside diameters of inner pipe having thickness 'δ' and thermal conductivity 'k'.

$$\delta = \frac{d_o - d_i}{2} = r_o - r_i \tag{8}$$

A_m is the mean peripheral area of inner pipe and is expressed as

$$A_m = \frac{A_o + A_i}{2} \tag{9}$$

$A_m = \pi\, d_m L$, $A_o = \pi\, d_o L$ and $A_i = \pi\, d_i L$, Therefore,

$$d_m = \frac{d_o + d_i}{2}$$

D_i and D_o are inside and outside diameters of outer pipe respectively.

Using equation (6), overall heat transfer coefficient, U for the arrangment can be expressed as

$$\frac{1}{UA_o} = \left(\frac{1}{h_i A_i} + \frac{\delta}{kA_m} + \frac{1}{h_o A_o} \right)$$

$$\frac{1}{U} = \left(\frac{A_o}{A_i}\frac{1}{h_i} + \frac{A_o}{A_m}\frac{\delta}{k} + \frac{1}{h_o} \right) \tag{10}$$

Substituting values of A_i , A_o and A_m in equation (10), we get

$$\frac{1}{U}=\left(\frac{d_o}{d_i}\frac{1}{h_i}+\frac{d_o}{d_m}\frac{d_o-d_1}{2}\frac{1}{k}+\frac{1}{h_o}\right) \tag{11}$$

Substituting value of d_m from equation (9) in equation (11), we get

$$\frac{1}{U}=\left(\frac{d_o}{d_i}\frac{1}{h_i}+\frac{2d_o}{d_o+d_i}\frac{d_o-d_1}{2}\frac{1}{k}+\frac{1}{h_o}\right)$$

$$\frac{1}{U}=\left(\frac{d_o}{d_i}\frac{1}{h_i}+\frac{d_o-d_1}{d_o+d_i}\frac{d_o}{k}+\frac{1}{h_o}\right) \tag{12}$$

Equation (12) represents overall heat transfer coefficient with reference to outer surface of inner pipe of a double pipe heat exchanger.

Heat Transfer Analysis in Heat Exchanger

Heat is transferred from hot fluid to cold fluid in heat exchanger and it involves the following steps

i) Heat lost by hot fluid is expressed as

$$Q_h = m_h\, c_{ph}\, (T_{hi} - T_{ho}) \tag{13}$$

ii) Heat gained by cold fluid is expressed as

$$Q_c = m_c\, c_{pc}\, (T_{co} - T_{ci}) \tag{14}$$

iii) Heat transfer in heat exchanger is expressed as

$$Q = U\, A\, (\Delta T)_m \tag{15}$$

From energy balance considerations, heat transferred in heat exchanger is equal to heat lost by hot fluid which is gained by cold fluid.

$$Q = Q_h = Q_c \tag{16}$$

Performance of a heat exchanger can be analyzed by using following methods

1. **Logarithmic Mean Temperature Difference or LMTD Method**
2. **Effectiveness of Number of Transfer Unit (NTU) Method**

1. Logrithmic Mean Temperature Difference (LMTD) Method:

LMTD method is used when temperatures of both the fluids at inlet and outlet of the heat exchanger are known. Performance analysis by this method is carried out by making following assumptions:

i) Overall heat transfer coefficient, U remains constant along the length of heat exchanger.

ii) Specific heats and mass flow rates of both the fluids remain constant.

iii) Heat exchanger is perfectly insulated and no loss of heat occurs.

iv) Axial conduction along the pipes is negligible.

A. Parallel Flow Heat Exchanger:

Consider a parallel flow heat exchanger of length 'L' as shown in Figure 3 and let m_h and m_c are mass flow rates of hot and cold fluids respectively.

T_{hi} and T_{ho} are temperatures of hot fluid at inlet and outlet of heat exchanger respectively.

T_{ci} and T_{co} are temperatures of cold fluid at inlet and outlet of heat exchanger respectively

c_{ph} and c_{pc} are specific heats of hot and cold fluids respectively.

θ_i and θ_o represent temperature difference between hot and cold fluids at inlet and outlet of heat exchanger respectively and are expressed as

$$\theta_i = T_{hi} - T_{ci} \text{ and } \theta o = T_{ho} - T_{co} \qquad (17)$$

C_h and C_c are heat capacity rates of hot and cold fluid respectively.

Heat capacity rate of a fluid is defined as amount of heat required to increase the temperature of a fluid by 1 °C and is expressed as

$$C_h = m_h C_{ph,}, C_c = m_c C_{pc} \qquad (18)$$

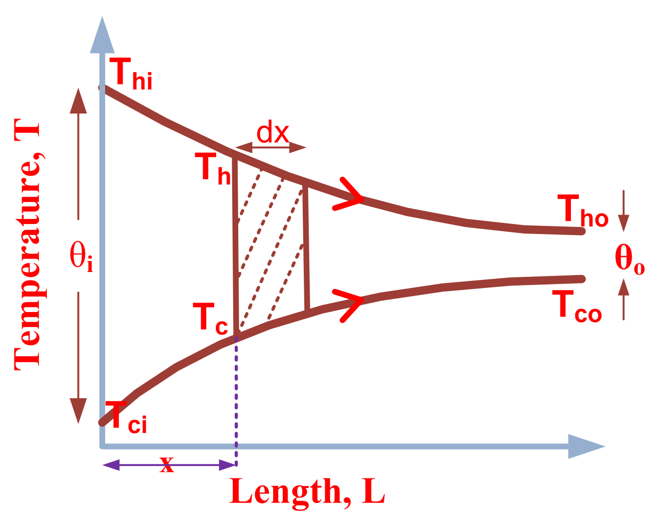

Figure 3

Consider an element of area dA and thickness dx at a distance 'x' from inlet of heat exchanger. Let T_h and T_c are temperatures of hot and cold fluid at inlet of the element and θ represents temperature difference of hot and cold fluid at inlet of the element.

$$\theta = T_h - T_c \qquad (19)$$

Let dT_h and dT_c represent change in temperature of hot and cold fluids respectively during the flow through the element. Let $d\theta$ be the difference in change in temperature of hot and cold fluids during flow through the element and is expressed as

$$d\theta = dT_h - dT_c \quad (20)$$

During flow through the element, heat transferred is equal to heat lost by hot fluid and heat gained by cold fluid.

If dQ is the amount of heat transferred during flow of fluids through the element then

$$dQ = UdA\,\theta = m_c C_{pc}\, dT_c = -m_h C_{ph}\, dTh \quad (21)$$

(-ve sign if temperature of fluid decreases along the length of heat exchanger)

Using equation (18), equation (21) can be written as

$$dQ = UdA\,\theta = C_c\, dT_c = -C_h\, dTh \quad (22)$$

From equation (22), we can write

$$dT_c = \frac{UdA\theta}{C_c},\ dT_h = -\frac{UdA\theta}{C_h} \quad (23)$$

Substituting the values of dT_c and dT_h from equation (23) in equation (20), we get

$$d\theta = -\frac{UdA\theta}{C_h} - \frac{UdA\theta}{C_c}$$

$$d\theta = -UdA\theta\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\frac{d\theta}{\theta} = -UdA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \quad (24)$$

Integrating equation (24) between limits θ_i and θ_o

$$\int_{\theta_i}^{\theta_o} \frac{d\theta}{\theta} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_0^A dA$$

$$\log_e\left(\frac{\theta_o}{\theta_i}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\log_e\left(\frac{\theta_i}{\theta_o}\right) = UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \quad (25)$$

Heat lost by hot fluid during flow through heat exchanger is expressed as

$$Q = m_h C_{ph} (T_{hi} - T_{ho})$$

Therefore, $C_h = \frac{Q}{\left(T_{hi} - T_{ho}\right)}$ (26)

Heat gained by cold fluid during flow through heat exchanger is expressed as

$Q = m_c c_{pc} (T_{co} - T_{ci})$

Therefore, $C_c = \frac{Q}{\left(T_{co} - T_{ci}\right)}$ (27)

Substituting values of C_h and C_c from equations (26) and (27) in equation (25), we get

$$\log_e\left(\frac{\theta_i}{\theta_o}\right) = UA\left(\frac{(T_{hi} - T_{ho})}{Q} + \frac{(T_{co} - T_{ci})}{Q}\right)$$

$$\log_e\left(\frac{\theta_i}{\theta_o}\right) = \frac{UA}{Q}\left[\left(T_{hi} - T_{ci}\right) - \left(T_{ho} - T_{co}\right)\right] \quad (28)$$

Using equation (17), equation (28) can be written as

$$\log_e\left(\frac{\theta_i}{\theta_o}\right) = \frac{UA}{Q}\left[\left(\theta_i\right) - \left(\theta_o\right)\right]$$

$$Q = UA\frac{\left[\left(\theta_i\right) - \left(\theta_o\right)\right]}{\log_e\left(\frac{\theta_i}{\theta_o}\right)} \quad (29)$$

Comparing equations (29) and (15), we can write

$$UA\left(\Delta T\right)_m = UA\frac{\left[\left(\theta_i\right) - \left(\theta_o\right)\right]}{\log\left(\frac{\theta_i}{\theta_o}\right)}$$

$$\left(\Delta T\right)_m = \frac{\left[\left(\theta_i\right) - \left(\theta_o\right)\right]}{\log_e\left(\frac{\theta_i}{\theta_o}\right)} \quad (30)$$

Equation 24.17 represents logarithmic mean temperature difference for a parallel flow heat exchanger.

REVIEW QUESTIONS:

Q.1 In a heat exchanger, heat transfer depends upon

a) Area of heat exchanger
b) Overall heat transfer coefficient
c) Logrithmic mean temperature difference
d) All the above

Q.2 Heat capacity rate of a fluid is equal to

a) Product of mass flow rate and specific heat of the fluid
b) Ratio of mass flow rate and specific heat of the fluid
c) Ratio of specific heat and mass flow rate of the fluid
d) All above

Q.3 Overall heat transfer coefficient of a plane wall is function of

a) Inside convective heat transfer coefficient, h_i
b) Outside cnvective heat transfer coefficient, h_o
c) Conductivity of wall material, k
d) All above

Q.4 LMTD of parallel flow heat exchanger is expressed as

a) $$\frac{\left[(\theta_i)-(\theta_o)\right]}{\log\left(\frac{\theta_i}{\theta_o}\right)}$$

b) $$\frac{\log\left(\frac{\theta_i}{\theta_o}\right)}{\left[(\theta_i)-(\theta_o)\right]}$$

c) $$\frac{\log\left(\frac{\theta_o}{\theta_i}\right)}{\left[(\theta_i)-(\theta_o)\right]}$$

d) None of the above