LESSON-27

B. **Counter flow Heat Exchanger:**

Consider a counter flow heat exchanger of length 'L' as shown in Figure 1 and let m_h and m_c are mass flow rates of hot and cold fluids respectively.

T_{hi} and T_{ho} are temperatures of hot fluid at inlet and outlet of heat exchanger respectively.

T_{ci} and T_{co} are temperatures of cold fluid at inlet and outlet of heat exchanger respectively

C_{ph} and C_{pc} are specific heats of hot and cold fluids respectively.

θ_i and θ_o represent temperature difference between hot and cold fluids at inlet and outlet of heat exchanger respectively and are expressed as

$$\theta_i = T_{hi} - T_{co} \text{ and } \theta_o = T_{ho} - T_{ci} \quad (1)$$

C_h and C_c are heat capacity rates of hot and cold fluid respectively.

Heat capacity rate of a fluid is defined as amount of heat required to increase temperature of a fluid by 1 °C and is expressed as

$$C_h = m_h C_{ph},\ C_c = m_c C_{pc} \quad (2)$$

Consider an element of area dA and thickness dx at a distance 'x' from inlet of heat exchanger. Let T_h and T_c are temperatures of hot and cold fluid at inlet of the element and θ represents temperature difference of hot and cold fluid at inlet of the element.

$$\theta = T_h - T_c \quad (3)$$

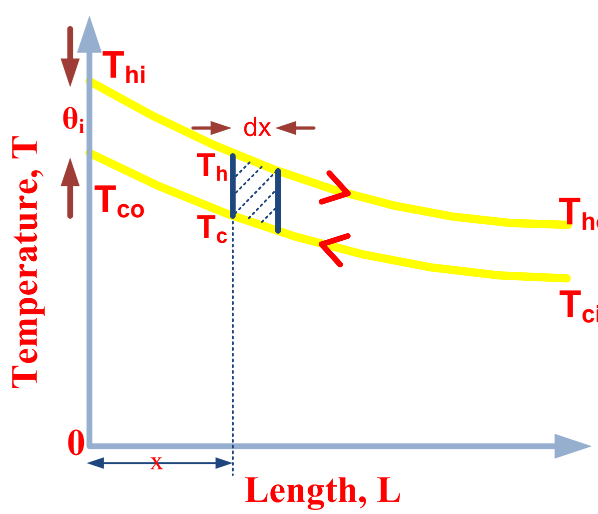


Figure 1

Let dT_h and dT_c represent change in temperature of hot and cold fluids respectively during the flow through the element. Let $d\theta$ be the difference in change in temperature of hot and cold fluids during flow through the element and is expressed as

$$d\theta = dT_h - dT_c \tag{4}$$

During flow through the element, heat transferred is equal to heat lost by hot fluid and heat gained by cold fluid.

If dQ is the amount of heat transferred during flow of fluids through the element then

$$dQ = UdA\,\theta = -m_c\, c_{pc}\, dT_c = -\, m_h\, c_{ph}\, dTh \tag{5}$$

(-ve sign if temperature of fluid decreases along the length of heat exchanger)

Using equation (2), equation (5) can be written as

$$dQ = UdA\,\theta = -C_c\, dT_c = -\, C_h\, dTh \tag{6}$$

From equation (6), we can write

$$dT_c = -\frac{UdA\theta}{C_c},\; dT_h = -\frac{UdA\theta}{C_h} \tag{7}$$

Substituting the values of dT_c and dT_h from equation (7) in equation (4), we get

$$d\theta = -\frac{UdA\theta}{C_h} + \frac{UdA\theta}{C_c}$$

$$d\theta = UdA\theta\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$\frac{d\theta}{\theta} = UdA\left(\frac{1}{C_c} - \frac{1}{C_h}\right) \tag{8}$$

Integrating equation (8) between limits θ_i and θ_o

$$\int_{\theta_i}^{\theta_o} \frac{d\theta}{\theta} = U\left(\frac{1}{C_c} - \frac{1}{C_h}\right)\int_0^A dA$$

$$\log_e\left(\frac{\theta_o}{\theta_i}\right) = UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right) \tag{9}$$

Heat lost by hot fluid during flow through heat exchanger is expressed as

$$Q = m_h\, C_{ph}\,(T_{hi} - T_{ho}) = C_h\,(T_{hi} - T_{ho})$$

$$C_h = \frac{Q}{(T_{hi} - T_{ho})} \tag{10}$$

Heat gained by hot fluid during flow through heat exchanger is expressed as

$Q = m_c\, c_{pc}\, (T_{co} - T_{ci}) = C_c\, (T_{co} - T_{ci})$

$$C_c = \frac{Q}{(T_{co} - T_{ci})} \qquad (11)$$

Substituting values of C_h and C_c from equations (10) and (11) in equation (9), we get

$$\log_e\left(\frac{\theta_o}{\theta_i}\right) = UA\left(\frac{(T_{co} - T_{ci})}{Q} - \frac{(T_{hi} - T_{ho})}{Q}\right)$$

$$\log_e\left(\frac{\theta_o}{\theta_i}\right) = \frac{UA}{Q}\left[(T_{ho} - T_{ci}) - (T_{hi} - T_{co})\right] \qquad (12)$$

Using equation (1), equation (12) can be written as

$$\log_e\left(\frac{\theta_o}{\theta_i}\right) = \frac{UA}{Q}\left[(\theta_o) - (\theta_i)\right]$$

$$Q = UA\frac{\left[(\theta_o) - (\theta_i)\right]}{\log_e\left(\frac{\theta_o}{\theta_i}\right)} \qquad (13)$$

We know that in a heat exchanger, heat transfer between hot and cold fluids can be expressed as

$$Q = U\, A\, (\Delta T)_m \qquad (14)$$

Comparing equations (13) and (14), we can write

$$UA(\Delta T)_m = UA\frac{\left[(\theta_o) - (\theta_i)\right]}{\log_e\left(\frac{\theta_o}{\theta_i}\right)}$$

$$(\Delta T)_m = \frac{\left[(\theta_o) - (\theta_i)\right]}{\log_e\left(\frac{\theta_o}{\theta_i}\right)}$$

$$(\Delta T)_m = \frac{\left[(\theta_i) - (\theta_o)\right]}{\log_e\left(\frac{\theta_i}{\theta_o}\right)} \qquad (15)$$

Equation (15) represents logarithmic mean temperature difference for a counter flow heat exchanger.

C. Cross Flow Heat Exchanger

In order to determine logrithmic mean temperature difference for a counter flow heat exchanger, following relationship is used.

$(LMTD)_{cross} = F \times (LMTD)_{counter}$

Where 'F' is a correction factor and is a function of geometry of heat exchanger and inlet as well as outlet temperatures of both the fluids. Values of correction factor for different arrangements of cross flow and multipass shell and tube heat exchangers are given in Figure 3.

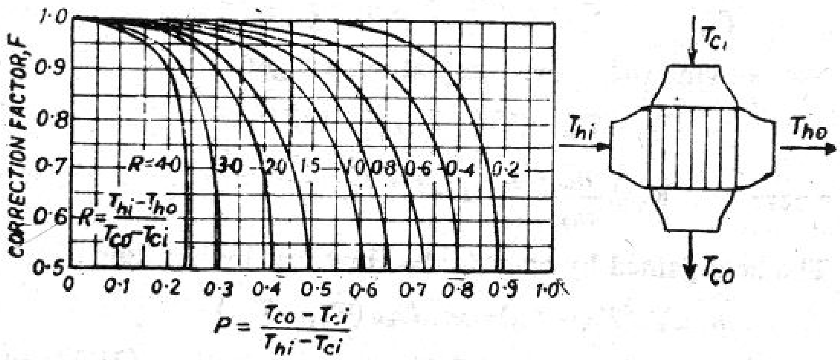
Fouling or Scaling

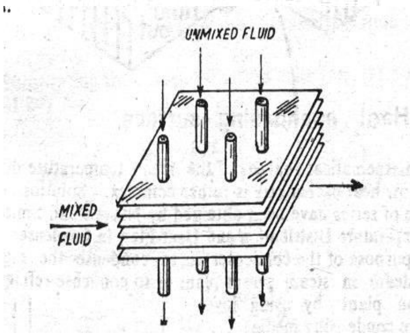
Heat transfer in a heat exchanger is reduced on account of accumulation of deposits on its heat transferring surface. The accumulated deopists on heat transferring surface of the heat exchanger increase thermal resistance resulting in decrease in heat transfer. The adverse effect of these deposits on the heat transfer is represented by a factor called fouling factor. Fouling factor in heat exchanger is denoted by R_f which represents increase in thermal resistance due to accumulation of solid doposits on heat transferring surface.

Fouling factor depends on the temperature and velocity of fluid as well as length of service of heat exhanger. Fouling factor value increases with increase in operating temperature and decrease in velocity of fluid. For new heat exchangers and clean heat transferring surfaces, fouling factor is zero. In order to determine the additional thermal resistance offered to heat transfer due to fouling or scaling, the following equation is used

$$R_f = \frac{1}{U_c} - \frac{1}{U_f}$$

Where U_c and U_f are the unit conductances of clean and fouled heat transferring surfaces repectively.





REVIEW QUESTIONS:

Q.1 In a counter flow heat exchanger, temperature differnce at the inlet of heat exchanger is equal to

a) Differnce of inlet temperatue of hot fluid and outlet temperature of cold fluid

b) Differnce of inlet temperatues of hot and cold fluids

c) Differnce of outlet temperatues of hot and cold fluids

d) None of the above

Q.2 In a counter flow heat exchanger, temperature differnce at the outlet of heat exchanger is equal to

a) Differnce of inlet temperatues of hot and cold fluids

b) Differnce of outlet temperatues of hot and cold fluids

c) Differnce of outlet temperatue of hot fluid and inlet temperature of cold fluid

d) None of the above

Q.3 LMTD of a cross flow heat exchanger is equal to

a) F X $(LMTD)_{counter}$

b) F X $(LMTD)_{Parallel}$

c) F / $(LMTD)_{Parallel}$

d) F / $(LMTD)_{counter}$

Q.4 Effect of flouling in heat exchangers is to

a) Increase heat transfer

b) Increase area of heat transfer

c) Decrease heat transfer

d) None of the above