LESSON 28

Effectiveness or Number of Transfer Units (NTU)

Logrithmic mean temperature difference method is very easy and simple to use for performance analysis of heat exchangers when inlet and outlet temperatures of hot and cold fluids are known or can be determined. However, if either temperature of the fluids is unknown or can not be determined, performance analysis by Logrithmic mean temperature difference method becomes cumbersome and complicated due to presence of logrithmic function. In such a case, effectiveness or number of transfer units (NTU) method is used for performance analysis of heat exchangers. Effectiveness or NTU method is based on the following concepts

- Capacity Ratio, C
- Effectiveness, ε
- Number of Transfer Units, NTU

Capacity ratio (C): It is defined as ratio of minimum to maximum capacity rate of fluids being used in a heat exchanger.

If $m_c\, c_{pc} < m_h\, c_{ph}$

Capacity ratio, $$C = \frac{m_c\, c_{pc}}{m_h\, c_{ph}} = \frac{c_{\min}}{c_{\max}} \qquad (1)$$

as $c_{min} = m_c\, c_{pc}$ and $c_{max} = m_h\, c_{ph}$

If $m_h\, c_{ph} < m_c\, c_{pc}$

Or $$C = \frac{m_h\, c_{ph}}{m_c\, c_{pc}} = \frac{c_{\min}}{c_{\max}} \qquad (2)$$

as $c_{min} = m_h\, c_{ph}$ and $c_{max} = m_c\, c_{pc}$

The fluid with lower heat capacity rate will undergo greater change in temperature as compared to fluid with higher heat capacity rate.

Effectiveness, ε:

Effectiveness of a heat exchanger is defined as ratio of actual heat transferred to maximum possible heat that can be transferred.

$$\varepsilon = \frac{Q_{actual}}{Q_{\max\ possible}} \qquad (3)$$

Q_{actual} = Heat lost by hot fluid = Heat gained by cold fluid

$Q_{actual} = m_h c_{ph} (T_{hi} - T_{ho}) = m_c c_{pc} (T_{co} - T_{ci})$ (4)

Maximum possible heat transfer will be achieved if a fluid undergoes a change in temperature equal to maximum temperature difference available between hot and cold fluids in the heat exchanger.

Maximum Temperature difference that exists in a heat exchanger = $T_{hi} - T_{ci}$

Maximum possible heat transfer occurs when a fluid of smaller heat capacity rate undergoes a change in temperature equal to maximum available temperature difference.

$Q_{maxpossibe} = C_{min} (T_{hi} - T_{ci})$ (5)

Substituting values of Q_{actual} and $Q_{max\ possible}$ from equations (4) and (5) in equation (3), we get

$$\varepsilon = \frac{m_h c_{ph} (T_{hi} - T_{ho})}{C_{min} (T_{hi} - T_{ci})} = \frac{C_h (T_{hi} - T_{ho})}{C_{min} (T_{hi} - T_{ci})} \quad (6)$$

or

$$\varepsilon = \frac{m_c c_{pc} (T_{co} - T_{ci})}{C_{min} (T_{hi} - T_{ci})} = \frac{C_c (T_{co} - T_{ci})}{C_{min} (T_{hi} - T_{ci})} \quad (7)$$

If $C_h < C_c$, then $C_h = C_{min}$, equation (6) can be expressed as

$$\varepsilon = \frac{C_{min} (T_{hi} - T_{ho})}{C_{min} (T_{hi} - T_{ci})} = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})} \quad (8)$$

If $C_c < C_h$, then $C_c = C_{min}$, equation (7) can be expressed as

$$\varepsilon = \frac{C_{min} (T_{co} - T_{ci})}{C_{min} (T_{hi} - T_{ci})} = \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})} \quad (9)$$

Effectiveness may also be defined as ratio of change in temperature of fluid with smaller capacity rate to maximum temperature difference existing in a heat exchanger.

Number of Transfer Units, NTU:

Number of transfer units is a dimensionless quantity and is an indicator of size of heat transferring areas of heat exchanger. Large value of NTU means larger heat transferring area. It is expressed as

$$NTU = \frac{UA}{C_{min}} \quad (10)$$

(a) Effectiveness for the Parallel Flow Heat Exchangers:

From definition of effectiveness of a heat exchanger, we can write

$$\varepsilon = \frac{C_h(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})} = \frac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})} \qquad (11)$$

Heat transferred in a heat exchanger can be expressed as

$Q = C_h(T_{hi} - T_{ho}) = C_c(T_{co} - T_{ci})$

Consider an element of area dA and thickness dx at a distance 'x' from inlet of a parallel flow heat exchanger as shown in Figure 1. Let dT_h and dT_c represent change in temperature of hot and cold fluids respectively during the flow through the element. Let $d\theta$ be the difference in change in temperature of hot and cold fluids during flow through the element and is expressed as

$$d\theta = dT_h - dT_c \qquad (12)$$

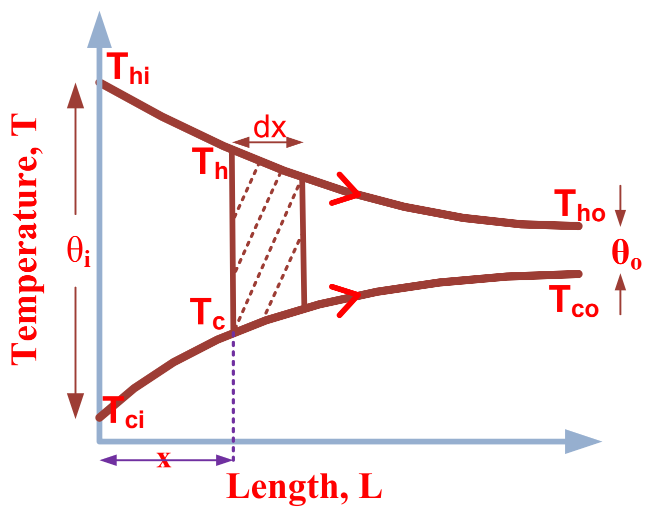

Figure 1

If dQ is the amount of heat transferred during flow of fluids through the element then

$$dQ = UdA\ \theta = C_c\ dT_c = -C_h\ dTh \qquad (13)$$

Substituting the values of dT_h and dT_c from equation (13) into the equation (12), we get

$$d\theta = -\frac{UdA\theta}{C_h} - \frac{UdA\theta}{C_c}$$

$$d\theta = -UdA\theta\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\frac{d\theta}{\theta} = -UdA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \tag{14}$$

Integrating equation (14) between limits θ_i and θ_o , we get

$$\int_{\theta_i}^{\theta_o} \frac{d\theta}{\theta} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_0^A dA$$

$\log_e\left(\frac{\theta_o}{\theta_i}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$ Taking anti log on both sides, we get

$$\left(\frac{\theta_o}{\theta_i}\right) = e^{-UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)} \tag{15}$$

For a parallel heat exchanger,

$\theta_i = T_{hi} - T_{ci}$ and $\theta o = T_{ho} - T_{co}$

Equation (15) can be written as

$$\left(\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}}\right) = e^{-UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)} \tag{16}$$

Using equation (11), values of T_{ho} and T_{co} can be written as

$$T_{ho} = T_{hi} - \frac{C_{\min}}{C_h}\left(T_{hi} - T_{ci}\right)\varepsilon \tag{17}$$

$$T_{co} = T_{ci} + \frac{C_{\min}}{C_h}\left(T_{hi} - T_{ci}\right)\varepsilon \tag{18}$$

Subtracting equation (18) from equation (17), we get

$$T_{ho} - T_{co} = \left(T_{hi} - T_{ci}\right) - C_{\min}\left(T_{hi} - T_{ci}\right)\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\varepsilon$$

$$T_{ho} - T_{co} = \left(T_{hi} - T_{ci}\right)\left(1 - \varepsilon\, C_{\min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\right)$$

$$\frac{\left(T_{ho} - T_{co}\right)}{\left(T_{hi} - T_{ci}\right)} = \left(1 - \varepsilon\, C_{\min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\right) \tag{19}$$

Using equation (19), equation (16) can be written as

$$1 - \varepsilon\, C_{\min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = e^{-UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)}$$

$$\varepsilon = \frac{1 - e^{-UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)}}{C_{\min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)} \qquad (20)$$

If $C_c < C_h$, then $C_c = C_{min}$ and $C_h = C_{max}$ equation (20) can be expressed as

$$\varepsilon = \frac{1 - e^{\frac{-UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)}}{\left(1 + \frac{C_{\min}}{C_{\max}}\right)} \qquad (21)$$

If $C_h < C_c$, then $C_h = C_{min}$ and $C_c = C_{max}$ equation (21) can be expressed as

$$\varepsilon = \frac{1 - e^{\frac{-UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)}}{\left(1 + \frac{C_{\min}}{C_{\max}}\right)} \qquad (22)$$

Using equations (2) and (10), equation (22) can be expressed as

$$\varepsilon = \frac{1 - e^{-NTU(1+C)}}{(1+C)} \qquad (23)$$

Effectiveness of a parallel flow heat exchanger can be determined from Figure 1 also.

The following two important cases are considered

i) Gas Turbine:

In case of a gas turbine $C = \frac{c_{\min}}{c_{\max}} \approx 1$, Equation (23) can be expressed as

$$\varepsilon = \frac{1 - e^{-2NTU}}{2} \qquad (24)$$

For a parallel flow heat exchanger, maximum value of effectiveness that can be achieved is 50% irrespective of values of its heat transferring area and overall heat transfer coefficient.

ii) Boiler and Condenser:

In case of a boiler and condenser, $C \frac{c_{\min}}{c_{\max}} \approx 0$, Equation (23) can be expressed as

$$\varepsilon = 1 - e^{-NTU} \qquad (25)$$

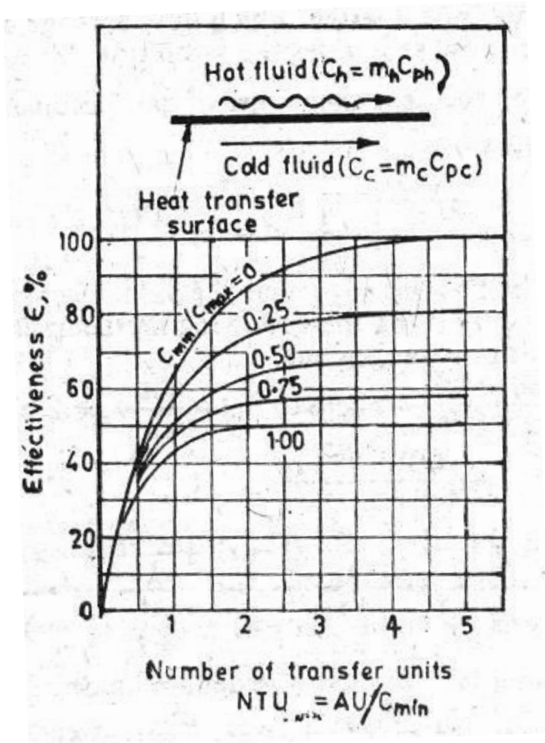

Figure 1 Effectiveness for Parallel Flow Heat Exchanger

REVIEW QUESTIONS:

Q.1 If in a heat exchanger, inlet and outlet temperatures of hot and cold fluids are not given or can not be determined, heat transfer analysis is generally carried out by using

a) LMTD Method

b) NTU Method

c) Any of the above mentioned methods

d) None of the above

Q.2 Number of transfer units (NTU) is an indicator of

a) Heat transferred in heat exchanger

b) Length of heat exchanger

c) Heat transferring area of heat exchanger

d) All above

Q.3 Effectiveness is defined as

a) Ratio of Q_{actual} to $Q_{maxpossible}$

b) Product of $Q_{maxpossible}$ and Q_{actual}

c) Ratio of $Q_{maxpossible}$ to Q_{actual}

d) None of the **above**

Q. 4 Maximum value of Effectiveness of parallel flow heat exchanger used in gas turbines can be

a) 50%

b) 100%

c) Less than 50%

d) Greater than 50% but less than 100%

Q.5 Capacity ratio is defined as

a) ratio of C_{min} to C_{max}

b) product of C_{min} and C_{max}

c) **ratio of C_{max} to C_{min}**

d) None of the above