LESSON-29

(b) Effectiveness for the Counter Flow Heat Exchangers:

From definition of effectiveness of a heat exchanger, we can write

$$\varepsilon = \frac{C_h(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})} = \frac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})} \qquad (1)$$

Heat transferred in a heat exchanger can be expressed as

$$Q = C_h(T_{hi} - T_{ho}) = C_c(T_{co} - T_{ci})$$

Consider an element of area dA and thickness dx at a distance 'x' from inlet of a counter flow heat exchanger as shown in Figure 1. Let dT_h and dT_c represent change in temperature of hot and cold fluids respectively during the flow through the element. Let $d\theta$ be the difference in change in temperature of hot and cold fluids during flow through the element and is expressed as

$$d\theta = dT_h - dT_c \qquad (2)$$

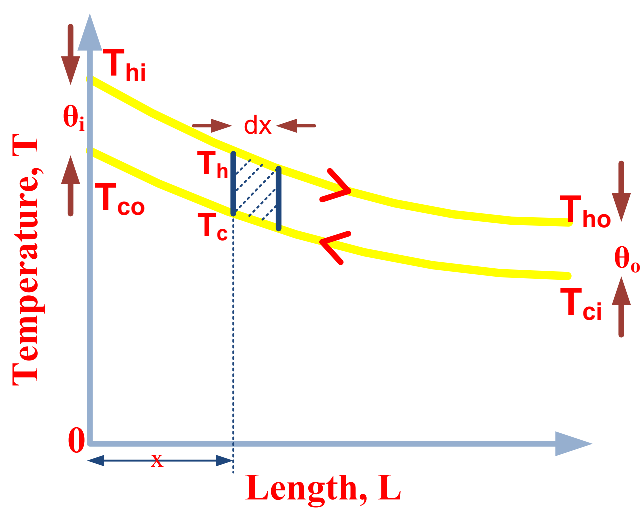

Figure 1

If dQ is the amount of heat transferred during flow of fluids through the element then

$$dQ = UdA\,\theta = -C_c\,dT_c = -C_h\,dTh \qquad (3)$$

Substituting the values of dT_h and dT_c from equation (4) into the equation (3), we get

$$d\theta = UdA\theta\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$\frac{d\theta}{\theta} = UdA\left(\frac{1}{C_c} - \frac{1}{C_h}\right) \qquad (5)$$

Integrating equation (5) between limits θ_i and θ_o

$$\int_{\theta_i}^{\theta_o} \frac{d\theta}{\theta} = U\left(\frac{1}{C_c} - \frac{1}{C_h}\right)\int_0^A dA$$

$$\log_e\left(\frac{\theta_o}{\theta_i}\right) = UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$\log_e\left(\frac{\theta_i}{\theta_o}\right) = -UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

Taking anti log on both sides, we get

$$\left(\frac{\theta_o}{\theta_i}\right) = e^{UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)}$$

$$\left(\frac{\theta_i}{\theta_o}\right) = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} \tag{6}$$

For a counter flow heat exchanger,

$\theta_i = T_{hi} - T_{co}$ and $\theta_o = T_{ho} - T_{ci}$

Equation (6) can be written as

$$\left(\frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}}\right) = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} \tag{7}$$

Using equation (1), values of T_{ho} and T_{co} can be written as

$$T_{ho} = T_{hi} - \varepsilon \frac{C_{\min}}{C_h}\left(T_{hi} - T_{ci}\right) \tag{8}$$

$$T_{co} = T_{ci} + \frac{C_{\min}}{C_h}\left(T_{hi} - T_{ci}\right)\varepsilon \tag{9}$$

Substituting the values of T_{ho} and T_{co} from equations (8) and (9) in equation (7), we get

$$\frac{\left(T_{hi} - T_{ci} - \varepsilon \frac{C_{\min}}{C_c}\left(T_{hi} - T_{ci}\right)\right)}{\left(T_{hi} - T_{ci} - \varepsilon \frac{C_{\min}}{C_h}\left(T_{hi} - T_{ci}\right)\right)} = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} \tag{10}$$

$$\frac{\left(T_{hi}-T_{ci}\right)\left(1-\varepsilon\frac{C_{\min}}{C_c}\right)}{\left(T_{hi}-T_{ci}\right)\left(1-\varepsilon\frac{C_{\min}}{C_h}\right)}=e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}$$

$$\frac{\left(1-\varepsilon\frac{C_{\min}}{C_c}\right)}{\left(1-\varepsilon\frac{C_{\min}}{C_h}\right)}=e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}$$

$$1-\varepsilon\frac{C_{\min}}{C_c}=e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}-e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}\varepsilon\frac{C_{\min}}{C_h}$$

$$1-e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}=\varepsilon\frac{C_{\min}}{C_c}-e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}\varepsilon\frac{C_{\min}}{C_h}$$

$$\varepsilon\left(\frac{C_{\min}}{C_c}-e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}\frac{C_{\min}}{C_h}\right)=1-e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}$$

$$\varepsilon=\frac{1-e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}}{C_{\min}\left(\frac{1}{C_c}-\frac{1}{C_h}e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}\right)} \qquad (11)$$

If $C_c < C_h$, then $C_c = C_{min}$ and $C_h = C_{max}$ equation (11) can be expressed as

$$\varepsilon=\frac{1-e^{-\frac{UA}{C_{\min}}\left(1-\frac{C_{\min}}{C_{\max}}\right)}}{\frac{C_{\min}}{C_{\min}}\left(1-\frac{C_{\min}}{C_{\max}}e^{-UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)}\right)}$$

$$\varepsilon=\frac{1-e^{-\frac{UA}{C_{\min}}\left(1-\frac{C_{\min}}{C_{\max}}\right)}}{\left(1-\frac{C_{\min}}{C_{\max}}e^{-\frac{UA}{C_{\min}}\left(1-\frac{C_{\min}}{C_{\max}}\right)}\right)}$$

$$\varepsilon=\frac{1-e^{-NTU(1-C)}}{\left(1-Ce^{-NTU(1-C)}\right)} \qquad (12)$$

The following two important cases are considered

i) Boiler and Condenser:

In case of a boiler and condenser, $C = \frac{c_{\min}}{c_{\max}} \approx 0$, Eqution (12) can be written as

$$\varepsilon = 1 - e^{-NTU}$$

ii) Gas Turbine:

In case of a gas turbine $C = \frac{c_{\min}}{c_{\max}} \approx 1$, Equation (12) becomes undeterminant and is solved as

Let $a = \frac{\mathrm{UA}}{\mathrm{C}_{\min}}$ and $x = \frac{c_{\min}}{c_{\max}}$

So, equation (12) can be written as

$$\varepsilon = \frac{1 - e^{-a(1-x)}}{\left(1 - xe^{-a(1-x)}\right)} \quad \text{or} \quad \varepsilon = \frac{1 - \frac{1}{e^{a(1-x)}}}{\left(1 - \frac{x}{e^{a(1-x)}}\right)} = \frac{\frac{e^{a(1-x)} - 1}{e^{a(1-x)}}}{\frac{e^{a(1-x)} - x}{e^{a(1-x)}}} \quad \text{or}$$

$$\varepsilon = \frac{e^{a(1-x)} - 1}{\left(e^{a(1-x)} - x\right)} \tag{13}$$

$$\Rightarrow \quad \varepsilon = \frac{1 + a(1-x) + \frac{a^2}{2!}(1-x)^2 + \frac{a^3}{3!}(1-x)^3 + \dots\dots\dots - 1}{1 + a(1-x) + \frac{a^2}{2!}(1-x)^2 + \frac{a^3}{3!}(1-x)^3 + \dots\dots\dots - x}$$

$$\Rightarrow \quad \varepsilon = \frac{a(1-x) + \frac{a^2}{2!}(1-x)^2 + \frac{a^3}{3!}(1-x)^3 + \dots\dots\dots}{1 - x + a(1-x) + \frac{a^2}{2!}(1-x)^2 + \frac{a^3}{3!}(1-x)^3 + \dots\dots\dots}$$

$$\Rightarrow \quad \varepsilon = \frac{(1-x)\left[a + \frac{a^2}{2!}(1-x) + \frac{a^3}{3!}(1-x)^2 + \dots\dots\dots\right]}{(1-x)\left(1 + a + \frac{a^2}{2!}(1-x) + \frac{a^3}{3!}(1-x)^2 + \dots\dots\dots\right)}$$

$$\Rightarrow \quad \varepsilon = \frac{a + \frac{a^2}{2!}(1-x) + \frac{a^3}{3!}(1-x)^2 + \dots\dots\dots}{\left(1 + a + \frac{a^2}{2!}(1-x) + \frac{a^3}{3!}(1-x)^2 + \dots\dots\dots\right)} \tag{14}$$

If $x = \frac{c_{\min}}{c_{\max}} \rightarrow 1$ then (1-x)=0, equation (14) can be written as

$$\varepsilon = \frac{a}{a+1} \quad \text{or} \quad \varepsilon = \frac{NTU}{NTU+1}$$

If the value of NTU = 1 then ε = 0.5 and heat exchanger becomes 50% efficient. It should be noted that the equations developed for the heat exchanger effectiveness are independent of the terminal temperatures of the fluid but are explicit functions of the dimensionless ratios NTU and ε. This enables one to plot ε verses NTU for selected values of C_{min}/C_{max}. Effectiveness of counter flow, Cross flow, One fluid mixed and other unmixed and Cross flow, both fluids mixed can be determined from Figures 2, 3 and 4 respectively.

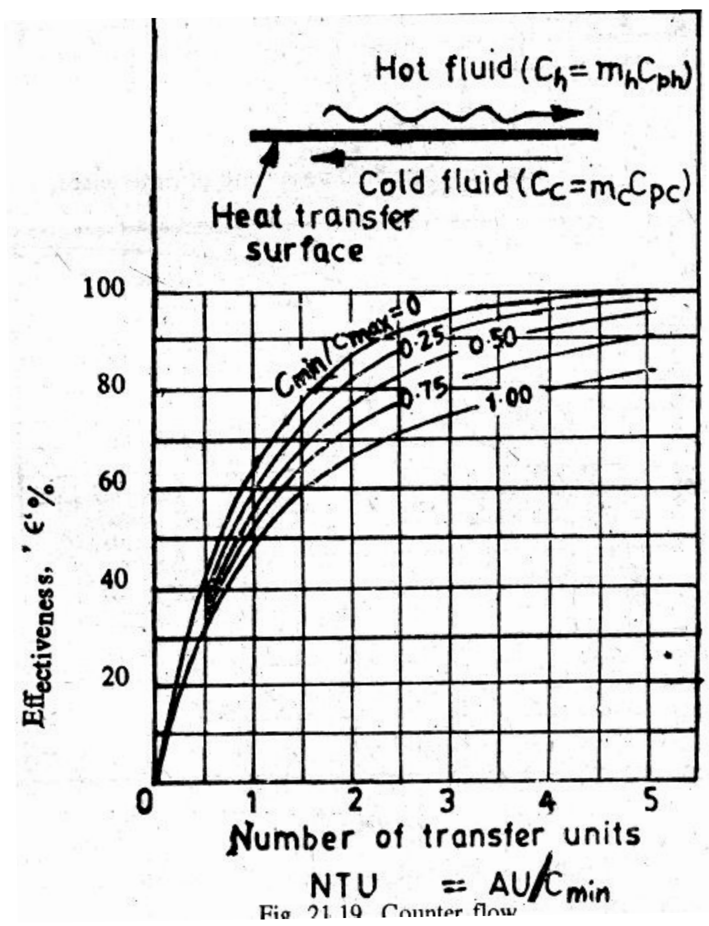

Figure 2 Counter Flow Heat exchanger

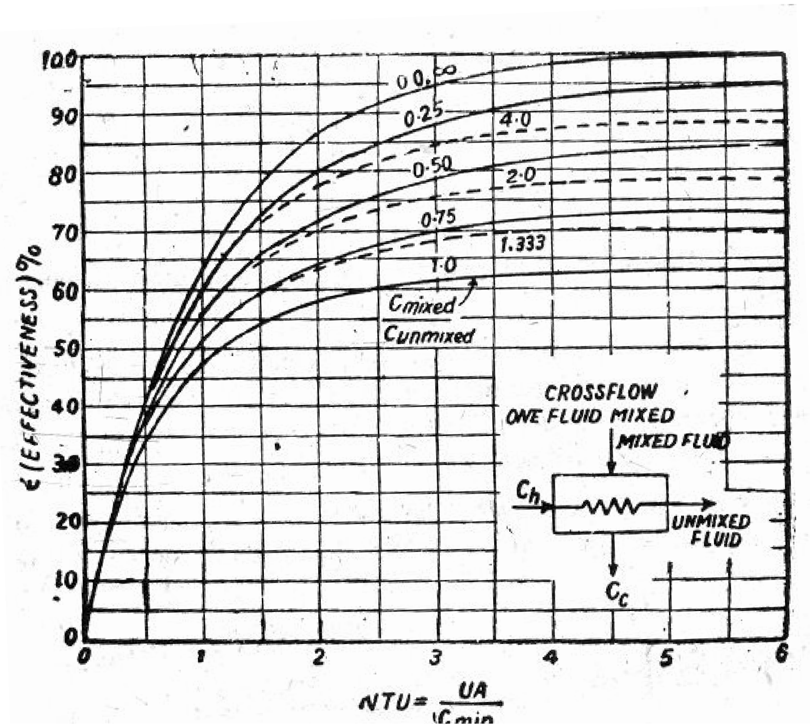

Figure 3 Cross flow. One fluid mixed and other unmixed.

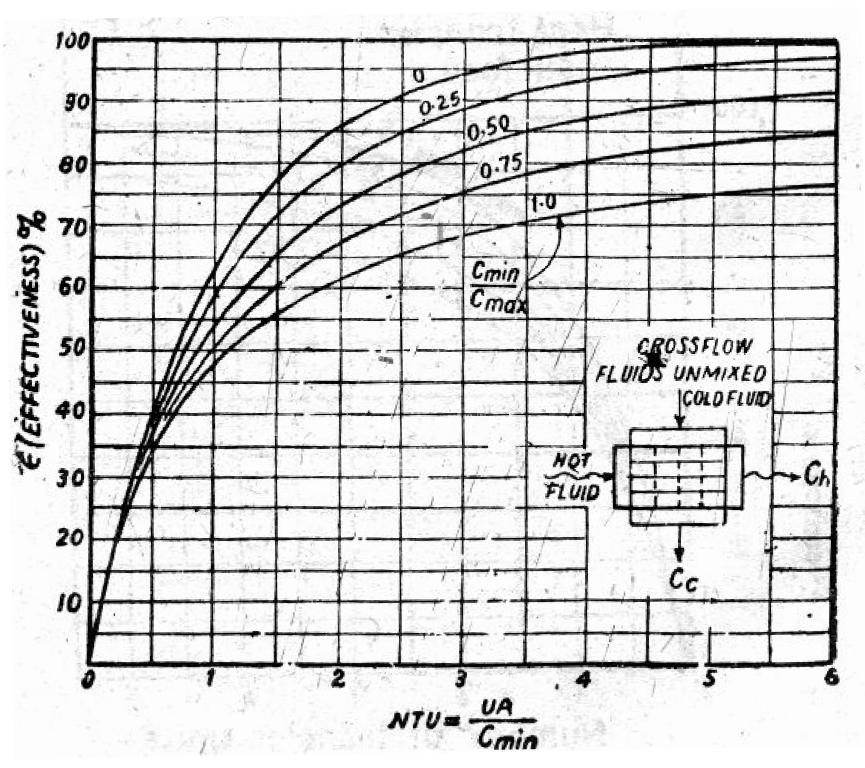

Figure 4 Cross flow both fluids mixed.

REVIEW QUESTIONS:

Q.1 If in a heat exchanger, inlet and outlet temperatures of hot and cold fluids are not given or can not be determined, heat transfer analysis is generally carried out by using

a) LMTD Method
b) NTU Method
c) Any of the above mentioned methods
d) None of the above

Q.2 Number of transfer units (NTU) is an indicator of

a) Heat transferred in heat exchanger
b) Length of heat exchanger
c) Heat transferring area of heat exchanger
d) All above

Q.3 Effectiveness is defined as

a) Ratio of Q_{actual} to $Q_{maxpossible}$
b) Product of $Q_{maxpossible}$ and Q_{actual}
c) Ratio of $Q_{maxpossible}$ to Q_{actual}
d) None of the **above**

Q. 4 Maximum value of Effectiveness of parallel flow heat exchanger used in gas turbines can be

a) 50%
b) 100%
c) Less than 50%
d) Greater than 50% but less than 100%

Q.5 Capacity ratio is defined as

a) ratio of C_{min} to C_{max}
b) product of C_{min} and C_{max}
c) **ratio of C_{max} to C_{min}**
d) None of the above