LESSON 3

One-Dimensional, Steady State Heat Conduction without Heat Generation:

i) Plane Wall or Slab of Uniform Conductivity without Heat Generation:

Consider steady state heat conduction through a plane wall of thickness 'L' and area 'A' having uniform conductivity 'k' as shown in Figure 1. Temperature on the left hand side of the wall is T_1 and on the right hand side it is T_2. Heat is flowing from left hand side to the right hand side as T_1 is greater than T_2. The general conduction equation which governs the conduction heat transfer is written as

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)+q_g\right]=\rho\, C\frac{\partial T}{\partial t} \qquad (1)$$

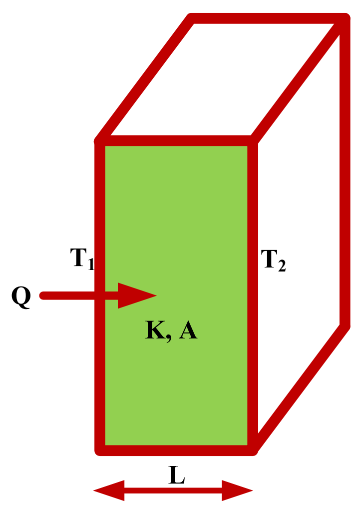

Figure 1

Since it is a case of one-dimensional, stead heat conduction through a wall of uniform conductivity without heat generation, therefore, $\frac{dT}{dt}=0$, $\frac{dT}{dy}=\frac{dT}{dz}=0$ and $q_g = 0$

Therefore, equation (1) reduces to

$$\frac{d^2T}{dx^2}=0 \qquad (2)$$

Equation (2) is used to determine the temperature distribution and heat transfer rate through the wall. Integrating equation (2) twice with respect to x, it can written as

$$T = C_1 x + C_2 \qquad (3)$$

Where, C_1 and C_2 are constants of integration.

Using the following boundary conditions:

i. At $x = 0$, $T = T_1$

Equation (3) is written as $C_2 = T_1$ (4)

ii. At $x = L$, $T = T_2$

Equation (3) can be written as $T_2 = C_1 L + C_2$

Or $T_2 = C_1 L + T_1$

$$C_1 = (T_2 - T_1)/L \qquad (5)$$

Substituting the values of C_1 and C_2 in equation (3)

$$T = \frac{T_2 - T_1}{L} x + T_1$$

Or $$T = T_1 - \frac{T_1 - T_2}{L} x \qquad (6)$$

Equation (6) represents temperature distribution in the wall. It means temperature at any point along the thickness of the wall can be obtained if values of temperatures T_1, T_2, thickness L and distance of the point form either of the faces of the wall are known.

Rate of heat transfer can be determined by using Fourier's law and can be expressed as

$$Q = -kA \frac{dT}{dx} \qquad (7)$$

Differentiating equation (6) with respect to x to obtain the expression for temperature gradient $\frac{dT}{dx}$

$$\frac{d}{dx} \int T = \frac{d}{dx} \int \left(T_1 - \frac{T_1 - T_2}{L} x \right)$$

$$\frac{dT}{dx} = -\frac{T_1 - T_2}{L} x$$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

Substituting the value of $\frac{dT}{dx}$ from above equation in equation (7), we get

$$Q = -k\,A\left(\frac{T_2 - T_1}{L}\right) \tag{8}$$

Equation (8) represents the heat transfer rate through the wall.

ii) Cylinder of Uniform Conductivity without Heat Generation:

Consider steady state heat conduction through a cylinder having r_1 and r_2 as inner and outer radii respectively and length 'L' as shown in Figure 2. Temperature of the inner and outer surfaces is T_1 and T_2 respectively. Heat is flowing from inner to outer surface as T_1 is greater than T_2. The general conduction equation which governs the conduction heat transfer is written as

$$\left[\frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \phi^2}\right) + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k}\right] = \frac{1}{\alpha}\ \frac{\partial T}{\partial t}dt \tag{9}$$

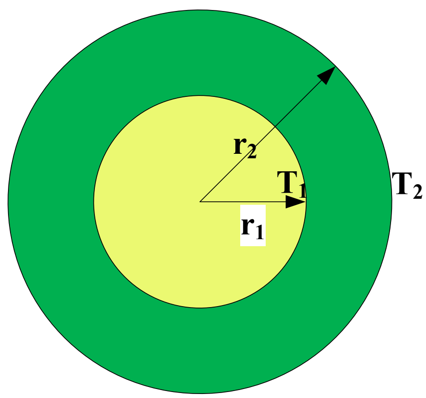

Figure 2

Since it is a case of one-dimensional, stead heat conduction through a wall of uniform conductivity without heat generation, therefore, $\frac{dT}{dt} = 0$, $\frac{dT}{d\phi} = \frac{dT}{dz} = 0$ and $q_g = 0$

Therefore, equation (9) reduces to

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

Or $$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0 \quad (10)$$

Equation (10) is used to determine the temperature distribution and heat transfer rate through the cylinder. Integrating equation (10) twice with respect to r, it can written as

$$r\frac{dT}{dr}=c_1 \text{ or } \frac{dT}{dr}=\frac{c_1}{r} \quad (11)$$

and $T = C_1 \log_e r + C_2$ (12)

Using the following boundary conditions:

i. At $r = r_1$, $T = T_1$

Equation (12) is written as $T_1 = C_1 \log_e r_1 + C_2$ (13)

ii. At $r = r_2$, $T = T_2$

Equation (12) can be written as

$$T_2 = C_1 \log_e r_2 + C_2 \quad (14)$$

Subtracting equation (14) from equation (13), we get

$$T_1 - T_2 = C_1 \log_e r_1 - C_1 \log_e r_2$$

$$T_1 - T_2 = C_1 \log_e \frac{r_1}{r_2}$$

$$C_1 = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \quad (15)$$

Substituting the values of C_1 from equation (15) in equation (13)

$$T_1 = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \log_e r_1 + C_2 \quad (16)$$

$$C_2 = T_1 - \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \log_e r_1$$

$$= \frac{T_1 \log_e r_1 - T_1 \log_e r_2 - T_1 \log_e r_1 + T_2 \log_e r_1}{\log_e \frac{r_1}{r_2}}$$

$$= \frac{T_2 \log_e r_1 - T_1 \log_e r_2}{\log_e \frac{r_1}{r_2}}$$

$$C_2 = \frac{T_1 \log_e r_2 - T_2 \log_e r_1}{\log_e \frac{r_2}{r_1}} \tag{17}$$

Substituting the values of C_1 and C_2 in equation (12), we get

$$T = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \log_e r + \frac{T_1 \log_e r_2 - T_2 \log_e r_1}{\log_e \frac{r_2}{r_1}}$$

$$T = \frac{1}{\log_e \frac{r_2}{r_1}} \left[T_1 \log_e r_2 - T_2 \log_e r_1 - (T_1 - T_2) \log_e r \right]$$

$$T = \frac{1}{\log_e \frac{r_2}{r_1}} \left[T_1 \log_e \frac{r_2}{r} - T_2 \log_e \frac{r}{r_1} \right] \tag{18}$$

Equation (18) represents temperature distribution in the cylinder. Rate of heat transfer can be determined by using Fourier's law and can be expressed as

$$Q = \left[-kA \frac{dT}{dr} \right]_{r=r_1} \tag{19}$$

$$Q = -k \times 2\pi r_1 \times L_1 \left(\frac{dT}{dr} \right)_{r=r_1} \tag{20}$$

From equation (11) we can write $\frac{dT}{dr} = \frac{c_1}{r}$

Substituting the value of C_1 from equation (15), we can write

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \times \frac{1}{r}$$

At $r = r_1$,

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \times \frac{1}{r_1} \tag{21}$$

Substituting the value of $\frac{dT}{dr}$ from equation (21) in equation (20), we get

$$Q = -k \times 2\pi r_1 \times L_1 \left(\frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \times \frac{1}{r_1} \right)$$

$$Q = -k \times 2\pi \times L_1 \left(\frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \right)$$

$$Q = \frac{2\pi k \times L_1 \left(T_1 - T_2 \right)}{\log_e \frac{r_2}{r_1}} \tag{22}$$

Equation (22) represents the heat transfer rate through the cylinder.

iii) Sphere of Uniform Conductivity without Heat Generation:

Consider steady state heat conduction through a hollow sphere having r_1 and r_2 as inner and outer radii respectively. Temperature of the inner and outer surfaces is T_1 and T_2 respectively. Heat is flowing from inner to outer surface as T_1 is greater than T_2. The general conduction equation which governs the conduction heat transfer is written as

$$\left[\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 T}{\partial\phi^2}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{q_g}{k}\right]=\frac{1}{\alpha}\ \frac{\partial T}{\partial t}dt$$

(22)

Since it is a case of one-dimensional, steady heat conduction through a shere without heat generation, therefore, $\frac{dT}{dt}=0$, $\frac{dT}{d\phi}=\frac{dT}{d\theta}=0$ and $q_g=0$

Therefore, equation (22) reduces to

$$\frac{d^2T}{dr^2}+\frac{2}{r}\frac{dT}{dr}=0 \tag{23}$$

Multiplying both sides of equation (23) by r^2 , we get

$$r^2\frac{d^2T}{dr^2}+2r\frac{dT}{dr}=0$$

Or

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \tag{24}$$

Equation (24) is used to determine the temperature distribution and heat transfer rate through the wall. Integrating equation (23) twice with respect to r, it can written as

$$r^2\frac{dT}{dr}=c_1 \text{ or } \frac{dT}{dr}=\frac{c_1}{r^2} \tag{25}$$

and

$$T=-\frac{C_1}{r}+C_2 \tag{26}$$

Using the following boundary conditions:

i. At $r = r_1$, $T = T_1$

Equation (26) is written as

$$T_1=-\frac{C_1}{r_1}+C_2 \tag{27}$$

ii. At $r = r_2$, $T = T_2$

Equation (26) can be written as

$$T_2 = -\frac{C_1}{r_2} + C_2 \tag{28}$$

Subtracting equation (28) from equation (27), we get

$$T_1 - T_2 = C_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$C_1 = \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \tag{29}$$

Substituting the values of C_1 from equation (29) in equation (27)

$$C_2 = T_1 + \frac{1}{r_1}\frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \tag{30}$$

Substituting the values of C_1 and C_2 from equations (29) and (30) in equation (26)

$$T = -\frac{1}{r}\frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} + T_1 + \frac{1}{r_1}\frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$T = T_1 - \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}\left(\frac{1}{r} - \frac{1}{r_1}\right) \tag{31}$$

Equation (31) represents temperature distribution in a sphere. Rate of heat transfer can be determined by using Fourier's law and can be expressed as

$$Q = \left[-kA\frac{dT}{dr}\right]_{r=r_1} \tag{32}$$

$$Q = -k \times 4\pi r^2 \left(\frac{dT}{dr}\right)_{r=r_1} \tag{33}$$

From equation (25) we can write $\frac{dT}{dr} = \frac{c_1}{r^2}$

Substituting the value of C_1 from equation (29), we can write

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \times \frac{1}{r^2}$$

At $r = r_1$

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \times \frac{1}{r_1^2} \qquad (34)$$

Substituting the value of $\frac{dT}{dr}$ from equation (34) from equation (33), we get

$$Q = -k \times 4\pi r^2 \times \frac{T_1 - T_2}{r^2} \; \frac{1}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$Q = \frac{4\pi r_1 r_2 k \left(T_1 - T_2\right)}{\left(r_2 - r_1\right)} \qquad (35)$$

Equation (35) represents the heat transfer rate through a sphere.

Heat Flow through Composite Geometries:

A) Composite Slab or Wall:

Consider a composite slab made of three different materials having conductivity k_1, k_2 and k_3, length L_1, L_2 and L_3 as shown in Figure 3. One side of the wall is exposed to a hot fluid having temperature T_f and on the other side is atmospheric air at temperature T_a. Convective heat transfer coefficient between the hot fluid and inside surface of wall is h_i (inside convective heat transfer coefficient) and h_o is the convective heat transfer coefficient between atmospheric air and outside surface of the wall (outside convective heat transfer coefficient). Temperatures at inner and outer surfaces of the composite wall are T_1 and T_4 whereas at the interface of the constituent materials of the slab are T_2 and T_3 respectively.

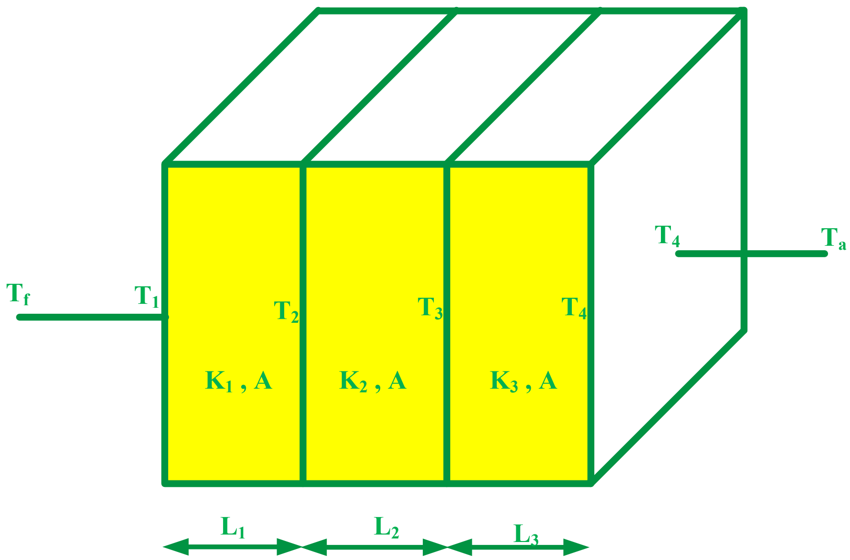

Figure 3

Heat is transferred from hot fluid to atmospheric air and involves following steps:

i) Heat transfer from hot fluid to inside surface of the composite wall by convection

$$Q = h_i\ A\left(T_f - T_1\right)$$

$$\frac{Q}{h_i\ A} = \left(T_f - T_1\right) \tag{36}$$

ii) Heat transfer from inside surface to first interface by conduction

$$Q = \frac{k_1 A\left(T_1 - T_2\right)}{L_1}$$

$$\frac{Q}{\frac{k_1 A}{L_1}} = \left(T_1 - T_2\right) \tag{37}$$

iii) Heat transfer from first interface to second interface by conduction

$$Q = \frac{k_2 A\left(T_2 - T_3\right)}{L_2}$$

$$\frac{Q}{\frac{k_2 A}{L_2}} = \left(T_2 - T_3\right) \tag{38}$$

iv) Heat transfer from second interface to outer surface of the composite wall by conduction

$$Q = \frac{k_3 A\left(T_3 - T_4\right)}{L_3}$$

$$\frac{Q}{\frac{k_3 A}{L_3}} = (T_3 - T_4) \tag{39}$$

v) Heat transfer from outer surface of composite wall to atmospheric air by convection

$$Q = h_o\, A(T_4 - T_a)$$

$$\frac{Q}{h_o\, A} = (T_4 - T_a) \tag{40}$$

Adding equations (36), (37), (38) and (40), we get

$$Q\left(\frac{1}{h_i\, A} + \frac{1}{\frac{k_1 A}{L_1}} + \frac{1}{\frac{k_2 A}{L_2}} + \frac{1}{\frac{k_3 A}{L_3}} + \frac{1}{h_o\, A}\right) = \left(T_f - T_1 + T_1 - T_2 + T_2 - T_3 + T_3 - T_4 + T_4 - T_a\right)$$

or

$$Q\left(\frac{1}{h_i\, A} + \frac{1}{\frac{k_1 A}{L_1}} + \frac{1}{\frac{k_2 A}{L_2}} + \frac{1}{\frac{k_3 A}{L_3}} + \frac{1}{h_o\, A}\right) = \left(T_f - T_a\right)$$

or

$$Q = \frac{\left(T_f - T_a\right)}{\frac{1}{h_i\, A} + \frac{1}{\frac{k_1 A}{L_1}} + \frac{1}{\frac{k_2 A}{L_2}} + \frac{1}{\frac{k_3 A}{L_3}} + \frac{1}{h_o\, A}} \tag{41}$$

If composite slab is made of 'n' number of materials, then equation (41) reduces to

$$Q = \frac{\left(T_f - T_a\right)}{\frac{1}{A}\left(\frac{1}{h_i} + \frac{1}{h_o} + \sum_{n=1}^{n=n}\left(\frac{L_n}{k_n}\right)\right)} \tag{42}$$

If inside and outside convective heat transfer coefficients are not to be considered, then equation (42) is expressed as

$$Q = \frac{\left(T_f - T_a\right)}{\frac{1}{A}\left(\sum_{n=1}^{n=n}\left(\frac{L_n}{k_n}\right)\right)} \tag{43}$$

B) Composite Cylinder:

Consider a composite cylinder consisting of inner and outer cylinders of radii r_1, r_2 and thermal conductivity k_1, k_2 respectively as shown in Figure 4. Length of the composite cylinder is L. Hot fluid at temperature T_f is flowing inside the composite cylinder. Temperature at the inner surface of the composite cylinder exposed to hot fluid is T_1 and outer surface of the composite cylinder is at temperature T_3 and is exposed to atmospheric air at temperature T_a. The interface temperature of the composite cylinder is T_2.. Convective heat transfer coefficient between the hot fluid and inside surface of composite cylinder is h_i (inside convective heat transfer coefficient) and h_o is the convective heat transfer coefficient between atmospheric air and outside surface of the composite cylinder (outside convective heat transfer coefficient). Heat is transferred from hot fluid to atmospheric air and involves following steps:

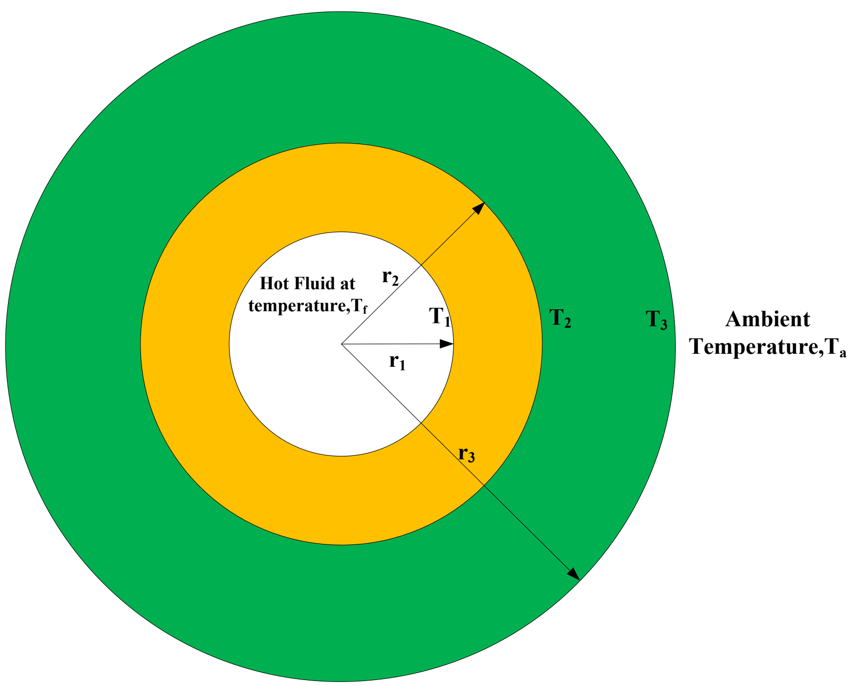

Figure 4

i) **Heat transfer from hot fluid to inside surface of the composite cylinder by convection**

$$Q = h_i\, A\left(T_f - T_1\right)$$

$$Q = h_i\, 2\pi r_1 L\left(T_f - T_1\right)$$

$$\frac{Q}{h_i\, 2\pi r_1 L} = \left(T_f - T_1\right) \tag{44}$$

ii) Heat transfer from inside surface to interface by conduction

$$Q = \frac{k_1 2\pi L\left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}}$$

$$\frac{Q}{\dfrac{k_1 2\pi L}{\log_e \frac{r_2}{r_1}}} = \left(T_1 - T_2\right) \tag{45}$$

iii) Heat transfer from interface to outer surface of the composite cylinder by conduction

$$Q = \frac{k_2 2\pi L\left(T_2 - T_3\right)}{\log_e \frac{r_3}{r_2}}$$

$$\frac{Q}{\dfrac{k_2 2\pi L}{\log_e \frac{r_3}{r_2}}} = \left(T_2 - T_3\right) \tag{46}$$

iv) Heat transfer from outer surface of composite wall to atmospheric air by convection

$$Q = h_o\, 2\pi r_3 L\left(T_4 - T_a\right)$$

$$\frac{Q}{h_o\, 2\pi r_3 L} = \left(T_4 - T_a\right) \tag{47}$$

Adding both sides of equations (44), (45),(46) and (47), we get

$$\frac{Q}{2\pi L}\left(\frac{1}{h_i r_1} + \frac{1}{\dfrac{k_1}{Log_e \frac{r_2}{r_1}}} + \frac{1}{\dfrac{k_2}{Log_e \frac{r_3}{r_2}}} + \frac{1}{h_o r_3}\right) = \left(T_f - T_1 + T_1 - T_2 + T_2 - T_3 + T_3 - T_a\right)$$

or

$$Q\left(\frac{1}{h_i r_1}+\frac{1}{\dfrac{k_1}{Log_e \dfrac{r_2}{r_1}}}+\frac{1}{\dfrac{k_2}{Log_e \dfrac{r_3}{r_2}}}+\frac{1}{h_o r_3}\right)=2\pi L\left(T_f-T_a\right)$$

or

$$Q=\frac{2\pi L\left(T_f-T_a\right)}{\left(\dfrac{1}{h_i r_1}+\dfrac{1}{\dfrac{k_1}{Log_e \dfrac{r_2}{r_1}}}+\dfrac{1}{\dfrac{k_2}{Log_e \dfrac{r_3}{r_2}}}+\dfrac{1}{h_o r_3}\right)} \qquad (48)$$

If the composite cylinder consists of 'n' cylinders, then equation (48) can be expressed as:

$$Q=\frac{2\pi L\left(T_f-T_a\right)}{\left(\dfrac{1}{h_i r_1}+\dfrac{1}{h_o r_{n+1}}+\sum_{n=1}^{n=n}\dfrac{1}{k_n}Log_e\left(\dfrac{r_{n+1}}{r_n}\right)\right)} \qquad (49)$$

If inside and outside convective heat transfer coefficients are not to be considered, then equation (3.41) is expressed as

$$Q=\frac{2\pi L\left(T_f-T_a\right)}{\left(\sum_{n=1}^{n=n}\dfrac{1}{k_n}Log_e\left(\dfrac{r_{n+1}}{r_n}\right)\right)} \qquad (50)$$

C) Composite Sphere:

Consider a composite sphere consisting of inner and outer cylinders of radii r_1, r_2 and thermal conductivity k_1, k_2 respectively. Hot fluid at temperature T_f is flowing inside the composite sphere. Temperature at the inner surface of the composite sphere exposed to hot fluid if T_1 and outer surface of the composite cylinder is at temperature T_3 and is exposed to atmospheric air at temperature T_a. The interface temperature of the composite cylinder is T_2. Convective heat transfer coefficient between the hot fluid and inside surface of composite sphere is h_i (inside convective heat transfer coefficient) and h_o is the convective heat transfer coefficient between atmospheric air and outside surface of the composite sphere (outside convective heat transfer coefficient). Heat is transferred from hot fluid to atmospheric air and involves following steps:

i) Heat transfer from hot fluid to inside surface of the composite sphere by convection

$$Q = h_i \, A\left(T_f - T_1\right)$$

$$Q = h_i \, 4\pi r_1^2 \left(T_f - T_1\right)$$

$$\frac{Q}{h_i \, 4\pi r_1^2} = \left(T_f - T_1\right) \tag{51}$$

ii) Heat transfer from inside surface to interface by conduction.

$$Q = \frac{4\pi k_1 r_1 r_2 \left(T_1 - T_2\right)}{r_2 - r_1}$$

$$\frac{Q\left(r_2 - r_1\right)}{4\pi k_1 r_1 r_2} = \left(T_1 - T_2\right) \tag{52}$$

iii) Heat transfer from interface to outer surface of the composite sphere by conduction

$$Q = \frac{4\pi k_2 r_2 r_3 \left(T_2 - T_3\right)}{r_3 - r_2}$$

$$\frac{Q\left(r_3 - r_2\right)}{4\pi k_2 r_2 r_3} = \left(T_2 - T_3\right) \tag{53}$$

iv) Heat transfer from outer surface of composite wall to atmospheric air by convection

$$Q = h_o \, 4\pi r_3^2 \left(T_4 - T_a\right)$$

$$\frac{Q}{h_o \, 4\pi r_3^2} = \left(T_4 - T_a\right) \tag{54}$$

Adding both sides of equations (51), (52),(53) and (54), we get

$$\frac{Q}{4\pi}\left(\frac{1}{h_i \, r_1^2} + \frac{1}{\frac{\left(r_2 - r_1\right)}{k_1 r_1 r_2}} + \frac{1}{\frac{\left(r_3 - r_2\right)}{k_2 r_2 r_3}} + \frac{1}{h_o \, r_3^2}\right) = \left(T_f - T_1 + T_1 - T_2 + T_2 - T_3 + T_3 - T_a\right)$$

or

$$Q\left(\frac{1}{h_i r_1^2}+\frac{1}{\frac{(r_2-r_1)}{k_1 r_1 r_2}}+\frac{1}{\frac{(r_3-r_2)}{k_2 r_2 r_3}}+\frac{1}{h_o r_3^2}\right)=4\pi\left(T_f-T_a\right)$$

or

$$Q=\frac{4\pi\left(T_f-T_a\right)}{\left(\frac{1}{h_i r_1^2}+\frac{1}{\frac{(r_2-r_1)}{k_1 r_1 r_2}}+\frac{1}{\frac{(r_3-r_2)}{k_2 r_2 r_3}}+\frac{1}{h_o r_3^2}\right)} \tag{54}$$

If the composite sphere consists of 'n' concentric spheres, then equation (54) can be expressed as:

$$Q=\frac{2\pi L\left(T_f-T_a\right)}{\left(\frac{1}{h_i r_1^2}+\frac{1}{h_o\left(r_{n+1}\right)^2}+\sum_{n=1}^{n=n}\left(\frac{r_{(n+1)}-r_n}{k_n r_n r_{(n+1)}}\right)\right)} \tag{55}$$

If inside and outside convective heat transfer coefficients are not to be considered, then equation (55) is expressed as

$$Q=\frac{4\pi\left(T_1-T_{n+1}\right)}{\left(\sum_{n=1}^{n=n}\left(\frac{r_{(n+1)}-r_n}{k_n r_n r_{(n+1)}}\right)\right)}$$

REVIEW QUESTIONS:

Q.1 If two surfaces of area A distance L apart, of a material having thermal conductivity k are at temperature T_1 and T_2, then heat flow rate through it will be

a) $\frac{kA}{L}(T_1 - T_2)$ b) $\frac{kL}{A}(T_1 - T_2)$

c) $\frac{k}{AL}(T_1 - T_2)$ d) $\frac{L}{kA}(T_1 - T_2)$

e) $\frac{A}{LK}(T_1 - T_2)$

Q.2 Two plane slabs of equal areas and conductivities in the ratio 1:2 are held together and temperature in between surface ends are t_1 and t_2 . If junction temperature in between two surfaces is desired to be $\frac{(T_1 + T_2)}{2}$, then their thickness should be in the ratio of

a) 1 : 2

b) 2 : 1

c) 1 : 1

d) 3 : 1

e) 1 : 3

Q.3 The heat flow rate through parallel walls of thickness L_1, L_2, L_3 and having surface areas A_1, A_2 and A_3, thermal conductivities k_1, k_2 and k_3, respectively and first and last walls maintained at temperatures T_1 and T_2 will be

a) $\frac{(T_1 - T_2)}{\frac{L_1}{A_1 k_1} + \frac{L_2}{A_2 k_2} + \frac{L_3}{A_3 k_3}}$

b) $\frac{(T_1 - T_2)}{\frac{k_1}{A_1 L_1} + \frac{k_2}{A_2 L_2} + \frac{k_3}{A_3 L_3}}$

c) $\frac{(T_1 - T_2)}{\frac{k_1 A_1}{L_1} + \frac{k_2 A_2}{L_2} + \frac{k_3 A_3}{L_3}}$

d) $\frac{(T_1 - T_2)}{\frac{L_1 A_1}{k_1} + \frac{L_2 A_2}{k_2} + \frac{L_3 A_3}{k_3}}$

e) $\frac{(T_1 - T_2)}{\frac{A_1}{L_1 k_1} + \frac{A_2}{L_2 k_2} + \frac{A_3}{L_3 k_3}}$

Q.4 If the inner and outer walls of a hollow sphere having surface areas of A_1 and A_2, and inner and outer radii r_1 and r_2 are maintained at temperatures T_1 and T_2, then rate of heat flow will be

a) $k\sqrt{A_1 A_2}\frac{(T_1 - T_2)}{r_2 - r_1}$

b) $\frac{k}{\sqrt{A_1 A_2}} \frac{(T_1 - T_2)}{r_1 - r_2}$

c) $4\pi k\frac{(T_1 - T_2)}{\sqrt{A_1 A_2}}$

d) $4\pi k r_1 r_2 \frac{(T_1 - T_2)}{\sqrt{A_1 A_2}}$

e) None of the above