LESSON-30

SOLVED EXAMPLES ON HEAT EXCHANGERS

Ex 30.1 **In a double pipe heat exchanger, oil is heated from -22 °C to -5 °C by water which enters and leaves the heat exchanger at 30 °C and 15 °C respectively. Flow rate of water through the heat exchanger is maintained at 7 kg/minute. Determine the heat exchanger area for parallel and counter flow arrangements if overall heat transfer coefficient is 860 W/ (m^2-K). Use LMTD method.**

Solution: **Given: Inlet temperature of cold fluid (oil), T_{ci}= -22 °C**

Outlet temperature of cold fluid (oil), T_{co}= -5 °C

Inlet temperature of hot fluid (water), T_{hi}= 30 °C

Outlet temperature of hot fluid (water), T_{ho}= 15 °C

Specific heat of hot fluid, C_{ph}= 4.186 kJ/(kg-K) = 4186 J/ (kg-K)

Mass flow rate of hot fluid, m_h = 7 kg/minute = 7/60 = 0.1166 kg/sec

Overall heat transfer coefficient, U = 860 W/(m^2-deg)

To determine: i) Heat exchanger area for parallel flow

ii) Heat exchanger area for counter flow

From energy balance considerations, heat transfer rate in heat exchanger is

Q = Heat lost by water (hot fluid) to oil (cold fluid)

$$Q = m\,C_{ph}\,(T_{hi} - T_{ho})$$

Q = 0.1166 X 4186 X (30-15) = 7321.31 J/sec

$$Q = UA(\Delta T)_m$$

$$A = Q / (U(\Delta T)_m)$$

$(\Delta T)_m$ is logrithmic mean temperature difference

i) For Parallel flow arrangement

$(\Delta T)_m$ for parallel flow arrangement is expressed as

$$(\Delta T)_m = \frac{[(\theta_i) - (\theta_o)]}{\log_e\left(\frac{\theta_i}{\theta_o}\right)}$$

For parallel flow arrangement;

$\theta_i = T_{hi} - T_{ci} = 30 - (-22) = 52$ °C

$\theta_i = T_{ho} - T_{co} = 15- (-5) = 20$ °C

$$Therefore, \Delta T_m = \frac{(\theta_i - \theta_o)}{log_e\left(\frac{\theta_i}{\theta_o}\right)} = \frac{(52-20)}{log_e\left(\frac{52}{20}\right)} = \frac{32}{log_e(2.6)} = 33.48$$

$$Therefore, A = \frac{Q}{U \times \Delta T_m} = \frac{7321.5}{860 \times 33.48} = 0.254\ m^2$$

ii) For counter flow arrangement

$(\Delta T)_m$ for counter flow arrangement is expressed as

$$(\Delta T)_m = \frac{[(\theta_i)-(\theta_o)]}{\log_e\left(\frac{\theta_i}{\theta_o}\right)}$$

For counter flow arrangement;

$\theta_i = T_{hi} - T_{co} = 30 - (-5) = 35\ ^oC$

$\theta_i = T_{ho} - T_{ci} = 15 - (-22) = 37\ ^oC$

$$Therefore, \Delta T_m = \frac{(\theta_i - \theta_o)}{log_e\left(\frac{\theta_i}{\theta_o}\right)} = \frac{(35-37)}{log_e\left(\frac{35}{37}\right)} = \frac{(-2\ )}{log_e(0.9459)}$$

$$= 35.99$$

$$Therefore, A = \frac{Q}{U \times \Delta T_m} = \frac{7321.5}{860 \times 35.99} = 0.236\ m^2$$

Ex 30.2 **Determine heat exchanger area for parallel and counter flow arrangements of a heat exchanger in which hot fluid is cooled from 350 ^oC to 175 ^{o} C by water enetering heat exchanger at 5 ^oC. Mass flow rate of hot fluid and water are 1000 kg/hr and 1250 k/hr respectively. Specific heat of hot fluid and water are 1.12 kJ/(kg-deg) and 4.186 kJ/(kg-deg) respectively. Overall heat transfer coefficient for both the arrangements of heat exchanger is 860 W/(m^2-K). Use LMTD method.**

Solution: **Given: Inlet temperature of cold fluid (water), T_{ci}= 5 ^oC**

Specific heat of cold fluid, C_{pc}= 4.186 kJ/(kg-K) = 4186 J/ (kg-K)

Mass flow rate of cold fluid, m_c = 1250 kg/hr = 1250/3600 = 0.3472 kg/sec

Inlet temperature of hot fluid, T_{hi}= 350 ^oC

Outlet temperature of hot fluid, T_{ho}= 175 ^oC

Specific heat of hot fluid, C_{ph}= 1.12 kJ/(kg-K) = 1120 J/ (kg-K)

Mass flow rate of hot fluid, m_h = 1000 kg/hr = 1000/3600 = 0.277 kg/sec

Overall heat transfer coefficient, U = 860 W/(m^2-deg)

To determine: i) Heat exchanger area for parallel flow

ii) Heat exchanger area for counter flow

Outlet temperature of cold fluid is not given, therefore, T_{co} is to be determined first. From energy balance considerations, heat transferred in heat exchanger is equal to heat lost by hot fluid which is gained by cold fluid. Therefore,

Heat transferred in heat exchanger = Heat lost by hot fluid = Heat gained by cold fluid

$$Q = Q_h = Q_c$$

$$Q_h = m_h \times C_{ph} \times (T_{hi} - T_{ho}) = 0.277 \times 1120 \times (350-175) = 54292 \text{ J/sec}$$

$$Q_c = m_c \times C_{pc} \times (T_{ci} - T_{co})$$

$$54292 = 0.3472 \times 4186 \times (T_{ci} - T_{co})$$

$$T_{co} = 5+37.35 = 42.35\ ^oC$$

Heat transfer rate in a heat exchanger is expressed as

$$Q = UA(\Delta T)_m$$
$$A = Q / (U(\Delta T)_m)$$

$(\Delta T)_m$ is logrithmic mean temperature difference

i) For Parallel flow arrangement

$(\Delta T)_m$ for parallel flow arrangement is expressed as

$$(\Delta T)_m = \frac{[(\theta_i)-(\theta_o)]}{\log_e\left(\frac{\theta_i}{\theta_o}\right)}$$

For parallel flow arrangement;

$\theta_i = T_{hi} - T_{ci} = 350 - 5 = 345\ ^oC$

$\theta_i = T_{ho} - T_{co} = 175- 42.35 = 132.65\ ^oC$

$$Therefore, \Delta T_m = \frac{(\theta_i - \theta_o)}{log_e\left(\frac{\theta_i}{\theta_o}\right)} = \frac{(345 - 132.65)}{log_e\left(\frac{345}{132.65}\right)} = \frac{212.35}{log_e(2.60)}$$
$$= 222.23$$

$$Therefore, A = \frac{Q}{U \times \Delta T_m} = \frac{54292}{860 \times 222.23} = 0.284\ m^2$$

ii) For counter flow arrangement

$(\Delta T)_m$ for counter flow arrangement is expressed as

$$(\Delta T)_m = \frac{[(\theta_i)-(\theta_o)]}{\log_e\left(\frac{\theta_i}{\theta_o}\right)}$$

For counter flow arrangement;

$\theta_i = T_{hi} - T_{co} = 350 - 42.35 = 307.65$ °C

$\theta_i = T_{ho} - T_{ci} = 175 - 5 = 170$ °C

$$Therefore, \Delta T_m = \frac{(\theta_i - \theta_o)}{log_e\left(\frac{\theta_i}{\theta_o}\right)} = \frac{(307.65 - 170)}{log_e\left(\frac{307.65}{170}\right)} = \frac{137.65}{log_e(1.8097)}$$

$$= 232.06$$

$$Therefore, A = \frac{Q}{U \times \Delta T_m} = \frac{54292}{860 \times 232.06} = 0.272\ m^2$$

Ex 30.3 **In a counter flow heat exchanger, air entering at a rate of 1 kg/sec is cooled from 55 ° C to 20 °C by water. Mass flow rate of water is 0.6 kg/sec and temperature of water at inlet of the heat exchanger is 10 °C. Using effectiveness method, determine the heat exchanger area required if the overall heat transfer coefficient is 30W/m²K.**

Solution: **Given: Inlet temperature of hot fluid (air), T_{hi}= 55 °C**

Outlet temperature of hot fluid (air), T_{ho}= 20 °C

Specific heat of hot fluid, C_{ph}= 1.004 kJ/kg-hr

Inlet temperature of cold fluid (water), T_{ci}= 10 °C

Outlet temperature of cold fluid (water), T_{co}= Not given

Specific heat of cold fluid, C_{pc}= 4.182 kJ/kg-deg

Mass flow rate of hot fluid (air), m_h = 1.0 kg/sec

Mass flow rate of cold fluid (water), m_c = 0.6 kg/sec

Overall heat transfer coefficient, U = 30 W/(m²-K)

To determine: i) Area of Heat exchanger for counter flow arrangement by using effectiveness method

Thermal capacity rates for the hot fluid (air) and cold fluid (water) are:

$$C_h = m_h c_h = 1.0 \times 1.005 = 1.005 \frac{KJ}{Sec-K} = \mathbf{1.005\ kW/K}$$

$$C_c = m_c c_c = 0.6 \times 4.186 = 2.115 \frac{kJ}{Sec - K} = \mathbf{2.511\ kW/K}$$

Therefore, C_c= C_{max} = 2.511 kW/K and C_h= C_{min} = 1.005 kW/K

Capacity ratio, $C = \frac{C_{min}}{C_{max}} = \frac{1.005}{2.511} = \mathbf{0.40}$

Effectiveness can be expressed as

$$\text{Effectiveness, } \epsilon = \frac{(t_{h1} - t_{h2})}{(t_{h1} - t_{c1})} = \frac{55-20}{55-10} = \mathbf{0.77}$$

$$\text{We know that Number of transfer units, } NTU = \frac{A \times U}{C_{min}}$$

$$\text{Therefore, } A = \frac{NTU \times C_{min}}{U}$$

i) Area of Heat exchanger for counter flow arrangement by using effectiveness method

The effectiveness for a counter flow arrangement is,

$$\epsilon = \frac{1 - \exp[-NTU(1-C)]}{1 - C\exp[-NTU(1-C)]}$$

$$\text{Rearranging: } \quad \frac{\epsilon - 1}{\epsilon C - 1} = \exp[-NTU(1-C)]$$

$$\frac{0.77-1}{0.77 \times 0.4 - 1} = \exp[-NTU(1-0.4)$$

$$\log_e \frac{0.23}{0.692} = -0.6\ NTU$$

$$-1.1015 = -0.6\ NTU \text{ or } NTU = \mathbf{1.83}$$

Therefore, the required transfer area is,

$$A = \frac{NTU \times C_{min}}{U} = \frac{1.83 \times 1005}{30} = \mathbf{61.30\ m^2}$$

Ex 30.4 **A fluid at 25 °C is used to cool an oil at 210 °C in an heat exchanger having 20 m^2. Oil is having specific heat of 2 kJ/(kg-K) and it flows at a rate of 2.72 kg/sec where as specific heat of cooling fluid is 4.5 kJ/(kg-K) and its mass flow rate is 0.88 kg/sec. For an overall heat transfer coefficient value of 315 W/(m^2-K), determine the outlet temperatures of oil and cooling fluid for parallel and counter flow arrangements using effectiveness Method.**

Solution: **Given: Inlet temperature of hot fluid (air), T_{hi}= 210 °C**

Specific heat of hot fluid, C_{ph}= 2.0 kJ/(kg-deg)

Inlet temperature of cold fluid (water), T_{ci}= 25 °C

Specific heat of cold fluid, C_{pc}= 4.5 kJ/(kg-deg)

Mass flow rate of hot fluid (oil), m_h = 2.72 kg/sec

Mass flow rate of cold fluid, m_c = 0.88 kg/sec

Overall heat transfer coefficient, U = 315 W/ (m^2-deg)

Heat transfer Area = 20 m^2

To determine: i) Outlet temperatures of hot and cold fluid using effectiveness method.

Solution: $C_h = m_h \times C_{ph} = 2.72 \times 2500 = \mathbf{5440 = C_{max}}$

$C_c = m_c \times C_{pc} = 0.88 \times 4500 = \mathbf{3960 = C_{min}}$

Heat lost by hot fluid = Heat gained by cold fluid

$$m_h C_{ph}(T_{hi} - T_{ho}) = m_c C_{pc}(T_{co} - T_{ci})$$

$$5440(210 - T_{ho}) = 3960(T_{co} - 25)$$

$$\mathbf{T_{co} = 313.33 - 1.373T_{h0}} \qquad (1)$$

This equation is valid for parallel as well as for counter flow.

$$\frac{C_{min}}{C_{max}} = \frac{3960}{5440} = \mathbf{0.73 = C}$$

$$\mathbf{NTU} = \frac{UA}{C_{min}} = \frac{315 \times 20}{3960} = \mathbf{1.59}$$

The value of NTU is also independent of flow direction.

(i) Parallel Flow

$$\epsilon = \frac{[1 - (e)^{-NTU(1+C)}]}{(1 + C)} = \frac{[1 - (e)^{-1.59 \times 1.73}]}{(1 + 0.73)} = \mathbf{0.54}$$

Effectiveness of a heat exchanger can also be expressed as

$$\epsilon = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}$$

$$\therefore \quad 0.55 = \frac{T_{co} - 25}{210 - 25}$$

$$\therefore \qquad T_{co} = 25 + 0.54(210 - 25) = \mathbf{125}℃$$

Using equation (1)

$$125 = 313.33 - 1.373T_{ho}$$

$$T_{h0} = \mathbf{137}℃$$

(ii) Counter Flow Arrangement

$$\in = \frac{1 - (e)^{-NTU(1-C)}}{1 - C(e)^{-NTU(1-C)}}$$

$$= \frac{1 - (e)^{-1.59\times0.27}}{1 - 0.73 \times (e)^{-1.59\times0.27}} = \mathbf{0.66}$$

$$\in = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}$$

$$\therefore T_{co} = T_{ci} + \in (T_{hi} - T_{ci})$$

T_{co} = 25+0.66 (210-25) = **147℃**

Again using the equation (1)

147 = 313.6-1.373T_{h0}

$$\therefore T_{ho} = \mathbf{121}℃$$