LESSON-32

Mass Transfer Coefficient:

Till now in our discussion, concentration gradient was considered to be the driving potential for transfer of mass. However, in practical situations involving fluids, convective mass transfer can not be neglected.

The governing equation for convective mass transfer is similar to the convective heat transfer equation and is expressed as

$$m_b = h_{mc} (C_{b1} - C_{b2}) \qquad (1)$$

where

m_b is is the **diffused mass** of component 'b'

h_{mc} is **mass transfer coefficient** of component 'b'

C_{b1} and C_{b2} are **mass concentrations** of component 'b'

For steady state, one dimensional diffusion of a fluid across layer of thickness ($X_2 - X_1$), mass diffusion can be expressed as

$$m_b = -DA\frac{(C_{b1}-C_{b2})}{(X_2-X_1)} \qquad (2)$$

Comparing equations (11) and (12), we get

$$h_{mc} = \frac{D}{(X_2-X_1)} \qquad (3)$$

Equation (3) represents **mass transfer coefficient** based on **concentration gradient**.

We know that mass flux or flow per unit area is represented as

$$\frac{m_b}{A} = -D_{bc}\frac{M_b}{GT}\frac{dp_b}{dx}$$, we can write

$$m_b = DA\frac{M_b}{GT}\frac{(p_{b1}-p_{b2})}{(X_2-X_1)} \qquad (4)$$

Using equation (3), equation (4) can be written as

$$\begin{aligned} m_b &= h_{mc}\frac{M_b}{GT}A(p_{b1}-p_{b2}) \\ &= h_{mp}A(p_{b1}-p_{b2}) \end{aligned} \qquad (5)$$

Where h_{mp} is **mass transfer coefficient** based on **pressure**

$$h_{mp} = h_{mc} M_b / GT$$

$$= h_{mc} / RT \qquad (6)$$

For diffusion of water vapor through a layer of stagnant air, mass diffusion for water is expressed as

$$
\begin{aligned}
m_w &= DA\frac{M_w p_t}{GT\left(X_2 - X_1\right)} loge \frac{(p_t - p_{w2})}{(p_t - p_{w1})} \qquad (7)\\
&= h_{mp}\, A(p_{w1} - p_{w2})
\end{aligned}
$$

Mass transfer coefficient, h_{mp}, based on **pressure difference** can be written as

$$h_{mp} = \frac{Dp_t}{\left(X_2 - X_1\right)(p_{w1} - p_{w2})}\frac{M_w}{GT} loge \frac{(p_t - p_{w2})}{(p_t - p_{w1})} \qquad (8)$$

Mass transfer coefficient, h_{mc}, based on **concentration gradient** can be expressed as

$$h_{mc} = \frac{Dp_t}{\left(X_2 - X_1\right)(p_{w1} - p_{w2})} loge \frac{(p_t - p_{w2})}{(p_t - p_{w1})} \qquad (9)$$

Reynolds Analogy:

The Reynolds analogy describes analogous behavior of mass, momentum and heat transfer and it was first recognized by Reynolds. The convective transport of mass, momentum and heat normally occur through a thin boundary layer close to the wall. The equations governing the transport of these quantities are analogous if the pressure gradient is equal to zero and the Prandtl Number (Pr) and Schmidt Number (Sc) are equal to unity. Under these conditions, their non-dimensional convective transport coefficients are related by the equation given below

$$\mathrm{Re}\frac{f}{2} = \mathrm{Nu} = \mathrm{Sh}$$

Or

$$\frac{f}{2}\frac{VL_c}{\nu} = \frac{h_{heat}L_c}{k} = \frac{h_{mass}L_c}{D_{bc}} \qquad (10)$$

where

f is friction factor

Re is Reynolds Number = $\frac{VL_c}{\nu}$

Nu is the Nusselt Number representing heat transfer = $\frac{h_{heat}L_c}{k}$

Sh the Sherwood Number representing mass transfer = $\frac{h_{mass}L_c}{D_{bc}}$

Equation (10) is known as the Reynolds analogy, and enables the calculation of heat transfer coefficient if either the friction factor or the mass transfer coefficient is known.

Example 14.1 Estimate the diffusion coefficient for ammonia in air at 25°C temperature and one atmospheric pressure.

For ammonia:

Molecular weight =20 and molecular volume = 25.81 cm^3/gm mole

For air:

Molecular weight = 26and molecular volume = 29.89 V cm^3/gm mole

Solution: The diffusion coefficient for binary gaseous mixtures is worked out from the relation:

$$D = 0.0043 \frac{T^{3/2}}{p_t\left(V_b^{1/3} + V_c^{1/3}\right)^2}\left(\frac{1}{M_b} + \frac{1}{M_c}\right)^{1/2}$$

Inserting the appropriate values in consistent units

$$D = 0.0043 \frac{(273 + 25)^{3/2}}{1 \times (25.81^{1/3} + 29.89^{1/3})^2}\left(\frac{1}{20} + \frac{1}{26}\right)^{1/2}$$

$$= 0.0043 \times \frac{5144.27}{(2.92+3.07)^2} \times (0.05 + 0.0385)^{1/2}$$

$$= 0.0043 \times \frac{5144.27}{35.88} \times 0.297 = 0.1831 \text{ cm}^2/\text{s}$$

Example 14.2 A rectangular system having steel walls of 8 mm thickness stores gaseous hydrogen at elevated pressure. The molar concentration of hydrogen in the steel at the steel at the inner and outer surfaces of the wall are approximated to be 1.0 kg-mol/m^3 and 0.0 kg-mol/m^3 respectively. Presuming that the binary diffusion coefficient for hydrogen in steel is 0.24×10^{-12} m^2/s, workout the diffusion flux for hydrogen through the steel wall. Point out the assumptions made in the derivation of the relation used by you.

Solution: The molar diffusion flux of hydrogen (h) through the steel wall (s) is prescribed

by the Fick's law

$$\frac{m_h}{A} = \frac{D_{hs}(C_{h1} - C_{h2})}{(x_2 - x_1)}$$

Which has been worked out with the following assumptions:

(i) Steady-state conditions

(ii) One-dimensional species diffusion through a plane wall which is approximately as a stationary medium.

(iii) No chemical reaction of the diffusing substance in the solid wall

Inserting the appropriate data in consistent units:

$$N_h = \frac{m_h}{A} = \frac{0.24\times10^{-12}(1-0)}{0.010} = 2.4 \times 10^{-11} \text{ kg-mol/s-m}^2$$

Since the molecular weight of hydrogen is 2 kg/kg-mol, the mass flux of hydrogen, is:

$$= 2 \times 2.4 \times 10^{-11} = 4.8 \text{ kg/s-m}^2$$

Example 14.3 A plastic membrane 0.25 mm thick has hydrogen gas maintained at pressures of 2.5 bar and 1 bar on its opposite sides. The binary diffusion coefficient of hydrogen in the plastic is 8.5×10^{-8} m^2/s and the solubility of hydrogen in the membrane is 1.5×10^{-3}kg-mol/ m^2-bar. Under uniform temperature conditions of 25°C, workout: (a) molar concentrations of hydrogen at the opposite faces of the membrane, (b) molar and mass diffusion flux of hydrogen through the membrane.

Solution: The molar concentrations (C), the partial pressures (p) and the solubility (S) of the diffusing gas are related to each other by the expression,

$$C = S\,p$$

Therefore molar concentrations of hydrogen at the opposite faces of the plastic membrane are:

$$C_{h1} = 1.5 \times 10^{-3} \times 2.5 = 3.5 \times 10^{-3} \text{ kg-mol/ m}^3\text{-bar}$$

$$C_{h1} = 1.5 \times 10^{-3} \times 2.5 = 3.5 \times 10^{-3} \text{ kg-mol/ m}^3\text{-bar}$$

(b) The molar diffusion flux of hydrogen through the membrane is worked out from the relation

$$N_h = \frac{m_h}{A} = \frac{D_{hp}(C_{h1}-C_{h2})}{(x_2-x_1)}$$

The subscripts h and p refer to hydrogen and plastic, respectively

Inserting appropriate values in consistent units,

$$N_h = \frac{8.5\times10^{-8}(3.75\times1.5\times10^{-3})}{0.5\times10^{-3}} = 38.25 \times 10^{-8} \text{kg-mol/s m}^2$$

Since the molecular weight of hydrogen is 2 kg/kg-mol, the mass flux of hydrogen is

$$= 2 \times 38.25 \times 10^{-8} = 76.5 \times 10^{-8} \text{ kg/ s-m}^2$$

Example14.4 Hydrogen gas at 1 bar and 400 k flows through a rubber tubing of 10mm inside radius and 20 mm outside radius. The diffusivity of hydrogen through rubber is stated to be 0.75×10^{-4} m^2/hr and the solubility of hydrogen is 0.052 m^3 of rubber t 1 atmosphere. What would be the diffusion loss of hydrogen per metre length of the rubber tubing? It may be presumed that resistance to diffusion of hydrogen from the outer surface of the tube is negligible.

Solution: The solubility of hydrogen at the operating pressure of 2 atmosphere is:

$$= 1 \times 0.052 = 0.052\ m^3/m^3 \text{ of rubber}$$

Then from the characteristic gas equation

$$pV = m.RT$$

Where the gas constant R for hydrogen is 4240 j/kg k

$$\therefore 1 \times 10^5 \times 0.052 = \mathrm{m} \times 4240 \times 400$$

$$\mathrm{m} = \frac{1\times10^5\times0.052}{4240\times400} = 0.003066 \text{kg/m}^3 \text{ of rubber}$$

Therefore, mass concentration of hydrogen at the inner surface of the pipe is 0.01635 kg/m^3. We can approximate the mass concentration to be zero at the pipe surface as resistance to diffusion is stated to be negligible at that surface. Thus

$$\mathrm{C_{h1}} = 0.31635 \text{ kg/m}^2 \text{ and } \mathrm{C_{h2}} = 0.0$$

The diffusion flux through a cylindrical system is given by

$$\mathrm{m} = \frac{\mathrm{D(C_{h1} - C_{h2})}}{\Delta \mathrm{x}} \mathrm{A_m}$$

$$\Delta \mathrm{x} = (\mathrm{r_2} - \mathrm{r_1}) = (20 - 10) \times 10^{-3} = 10 \times 10^{-3} \mathrm{m}$$

$$\mathrm{A_m} = \frac{2\pi \mathrm{l}(\mathrm{r_2}-\mathrm{r_1})}{\log_e(\mathrm{r_2}/\mathrm{r_1})} = \frac{2\pi\times1\times10\times10^{-3}}{\log_e(20/10)} = 9.06 \times 10^{-2} \mathrm{m}^2$$

$$\therefore \mathrm{m} = \frac{0.75\times10^{-4}\times(0.003066-0.0)}{10\times10^{-3}} \times (9.06 \times 10^{-2}) = 2.20 \times 10^{-5} \text{kg/hr}$$

Example14.5 The air pressure in a tyre tube of surface area 1 ∴m^2 and wall thickness of 0.01m is approximated to drop from 2 bar in a period of 5-days. The solubility of air in rubber is 0.07m^3 of rubber at 1 bar. Estimate the diffusivity of air in rubber at the operating temperature of 200 k if the volume of air in the tube is 0.025m^3.

Solution: Initial mass of air in the tube,

$$\mathrm{m_i} = \frac{\mathrm{P_i V}}{\mathrm{RT}} = \frac{2\times10^5\times0.025}{287\times200} = 0.08711 \text{kg}$$

Final mass of air in the tube

$$\mathrm{m_f} = \frac{\mathrm{P_f V}}{\mathrm{RT}} = \frac{1.99\times10^5\times0.025}{287\times200} = 0.08667 \text{ kg}$$

Mass of air escaped is equal to

$$m_i - m_f = 0.08711 - 0.086677 = 2.9 \times 10^{4}\text{kg}$$

Therefore, the mass flux equals

$$\frac{m_a}{A} = \frac{\text{mass escaped}}{\text{timeelapsed} \times \text{area}}$$

$$= \frac{2.9 \times 10^{-4}}{(5 \times 24 \times 3600) \times 1} = 0.662 \times 10^{-9} \text{ kg/ s-m}^2$$

The solubility of air at the mean operating pressure of (2+1.99)/2=1.995 bar is

$$= 1.995 \times 0.07 = 0.1396 \text{ m}^3 \text{ of air/m}^3 \text{ of rubber}$$

The air which escapes to atmosphere will be at 1 bar pressure and its solubility will remain at 0.07 m^3 of rubber.

The corresponding mass concentrations are worked out from the characteristic equation,

$$C_{a1} = \frac{PV}{RT} = \frac{1.995 \times 10^5 \times 0.1396}{287 \times 200} = 0.4852 \text{ kg/m}^3$$

$$C_{a2} = \frac{1 \times 10^5 \times 0.07}{287 \times 200} = 0.12195 \text{ kg/m}^3$$

The diffusion flux of air through rubber is:

$$\frac{m_a}{A} = \frac{D_a(C_{a1} - C_{a2}}{(x_2 - x_1)}$$

$$\therefore 1.342 \times 10^{-9} = \frac{D_a(0.4852 - 0.12195)}{0.01}$$

Therefore, diffusivity of air in rubber is

$$D_a = \frac{1.342 \times 10^{-9} \times 0.01}{(0.4852 - 0.12195)} = 3.67 \times 10^{-11} \text{ m}^2\text{/s}$$

Example14.6 **A distillation column containing a mixture of benzene and toluene is at a pressure of 1 atmosphere and 50°C temperature. The liquid and vapour phases contain 30 mol% and 45 mol % of benzene. At 50°C temperature, the vapour pressure of toluene is 70kN/m^2 and the diffusivity is 5×10^{-6} m^2/s. workout the rate of interchange of benzene and toluene between the liquid and vapour phases if resistance to mass lies in film 0.50 mm thick.**

Take atmospheric pressure=101 kN/m^2 and Universal gas constant G=8.314 kj/kg-mol K.

Solution: Let subscripts b and t refer to benzene and toluene respectively. At the liquid plane 1, the partial pressure of toluene is

$$p_{t_1} = \text{molar concentration} \times \text{vapour pressure}$$

$$= (1 - 0.3) \times 70 = 49 \text{ kN/m}^2$$

For the equi-molal diffusion, the molar diffusion flux of toluene is;

$$N_t = \frac{m_1}{A} = \frac{D}{GT}\frac{(P_{t_1} - P_{t_2})}{(x_2 - x_1)}$$

$$= \frac{5\times10^{-6}}{8.314\times323} \times \frac{(49-55)}{0.25\times10^{-3}} = -44.68 \times 10^{-6}$$ kg-mol/s-m^2

The negative sign indicates that transfer of toluene is from vapour to liquid. Benzene will diffuse in the opposite direction at the same rate.

Example14.7 Derive an expression for the steady state diffusion of a gas A through another stagnant gas B.

Oxygen is diffusing through stagnant carbon monoxide at 0°C and 800 mm of Hg pressure under steady state conditions. The partial pressure of oxygen at two planes 0.6 cm apart is 100 mm of Hg and 20 mm of Hg respectively. Calculate the rate of diffusion of oxygen in gm-moles through cm^2 area. It may be presumed that:

Diffusivity of oxygen in carbon monoxide =0.185 cm^2/s
Gas constant R=82.06 cm^2 atm/gm mole K.

Solution: Let subscripts 0 and c refer to oxygen and carbon monoxide.

Partial pressures of oxygen on the given planes are:

$$p_{0_1} = \frac{100}{800} = 0.125 \text{ atm}$$

$$p_{0_2} = \frac{20}{760} = 0.0263 \text{ atm}$$

Partial pressures of carbon monoxide are:

$$p_{c_1} = 1 - 0.125 = 0.875 \text{ atm}$$

$$p_{c_2} = 1 - 0.0263 = 0.9737 \text{ atm}$$

Log mean partial pressure for non-diffusing carbon monoxide is given by:

$$\text{LMPC} = \frac{p_{c_1} - p_{c_2}}{\log_e(p_{c_1}/p_{c_2})}$$

$$= \frac{0.875 - 0.9737}{\log_e(0.875/0.9737)} = 2.1264 \text{ atm}$$

The diffusion rate of oxygen is given by,

$$N_{0_2} = \frac{DA}{GT} \times \frac{p_t}{(x_2 - x_1)} \times \left(\frac{p_{0_1} - p_{0_2}}{\text{LMPC}}\right)$$

$$= \frac{0.185\times1}{82.06\times273} \times \frac{1}{0.6} \frac{(0.1125-0.0263)}{2.1264} = 5.579 \times 10^{-7}$$ gm-mol/s

Alternatively

$$N_{0_2} = \frac{DA}{GT} \times \frac{p_t}{(x_2-x_1)} \times \log_e \frac{p_{c_1}}{p_{c_2}}$$

$$= \frac{0.185 \times 1}{82.06 \times 273} \times \frac{1}{0.6} \times \log_e \frac{0.9737}{0.875} = 6.388 \times 10^{-7}$$ gm-mol/s

Example14.8 Estimate the diffusion coefficient of carbon tetrachloride into air from the following data recorded in a Stefan-tube experiment with carbon tetrachloride and oxygen:

Diameter of tube and its length above liqid surface: 1 cm and 15 cm respectively
Temperature and pressure maintained: 0°C and 760 mm of mercury
Evaporation of carbon tetrachloride: 0.03 gm in 12 hour
Vapour pressure of carbon tetrachloride: 30 mm of mercury

Solution: The molecular weights of carbon tetrachloride M_c and oxygen M_0 are:

$M_c = 12+4(31.5) = 154$ and $M_0 = 32$

Partial pressures of these elements at bottom and top are:

$p_{c_1} = 30$ mm of Hg ; $p_{c_2} = 0$
$p_{0_1} = 760 – 30 = 730$ mm of Hg
$p_{0_2} = 760$ mm of Hg and $p_t = 1 \times 10^5$ N/m^2

Further, $m = \frac{0.03 \times 10^{-10}}{12 \times 3600} = 6.9416 \times 10^{-10}$ kg/s

$$\frac{8.333 \times 10^{-10}}{154} = 4.507 \times 10^{-12} \text{ kg-mol/s}$$

$$A = \frac{\pi}{4}(0.01)^2 = 7.854 \times 10^{-5} \text{m}^2$$

$$(x_2 - x_1) = 15\ cm = 0.15m$$

Inserting these values in the diffusion equation,

$$m = \frac{DA}{GT} \frac{p_t}{(x_2 - x_1)} \log_e \left(\frac{p_{0_2}}{p_{0_1}}\right)$$

$$4.507 \times 10^{-12} = \frac{D \times 7.854 \times 10^{-5}}{8314} \times \frac{10^5}{0.15} \times \log_e \left(\frac{760}{730}\right) = 1.1015 \times 10^{-4} D$$

∴ Diffusion coefficient of carbon tetrachloride in air

$$D = \frac{4.507 \times 10^{-12}}{1.1015 \times 10^{-4}} = 4.0917 \times 10^{-8} \text{ m}^2/\text{s}$$

Example14.9 An open tank, 10 mm in diameter, contains 1 mm deep layer of benzene (Mol wt =78) bottom. The vapour pressure of benzene in the tank is 13.15 kN/m^2 and its diffusion takes place through a stagnant air film 2.5 thick, At the operating temperature of 20°C, the diffusivity of benzene in the tank is 8.0×10^{-6} m^2/s. If the benzene has a density of 880 kg/m^3, calculate the time taken for the entire benzene to evaporate. Take atmospheric pressure as 101.3 kN/m^2 and neglect any resistance to diffusion of beneze beyond the air film.

Solution: Let subscripts b and a refer to benzene and air respectively.

Partial pressures of benzene and air at the two levels are:

$p_{b_1} = 13.5$ kN/m^2 and $p_{b_2} = 0.0$ kN/m^2

$p_{a_1} = p_t - p_{b_1} = 101.3 - 13.5 = 87.8$ kN/m^2

$p_{a_2} = p_t - p_{b_2} = 101.3 - 0.0 = 101.3$ kN/m^2

The diffusion rate of benzene is given by

$$N_b = \frac{DA}{GT}\frac{p_t}{(x_2 - x_1)}\log_e\left(\frac{p_{a_2}}{p_{a_1}}\right)$$

$$= \frac{8.0\times 10^{-6}\left[\pi/4\times(6)^2\right]}{8.314\times 293}\times\frac{101.3}{0.0025}\times\log_e\frac{101.3}{87.8}$$

$$= 5.378\times 10^{-4}\text{kg}-\frac{\text{mol}}{\text{s}} = (5.378\times 10^{-4})\times 78 = 0.04195\text{kg/s}$$

Mass of benzene which is to be evaporated,

$$= \frac{\pi}{4}(10)^2\times 0.001\times 880 = 69.083\text{ kg}$$

∴ Time needed for evaporation= $\frac{69.083}{0.04195}$ =**1646.79s**