LESSON 4

One-Dimensional, Steady State Heat Conduction with Heat Generation:

One dimensional, steady state heat conduction is considered for following geometries

1) Slab
2) Cylinder
3) Sphere

1) One-Dimensional Heat Flow through a slab with Heat Generation:

i) When Temperature of Both Sides of Slab is Same:

Consider a slab of thickness 'L' and cross-sectional area 'A' through which heat flow takes place in x-direction. A heat source located at the center of the slab is generating 'q_g' amount of heat per unit volume per unit time as shown in Figure 1.

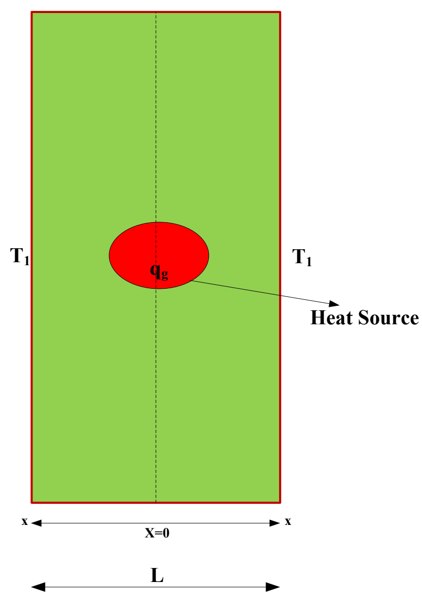

Figure 1

Heat generated is conducted equally towards the sides of the slab through a distance 'x' measured from center of the slab along x-direction. Temperature of both sides of the slab is same and is equal to T_1 as same amount of heat is flowing from the center towards the sides of the slab.

At the center of the slab x=0 and at the sides of the slab x= L/2. The general conduction equation under the given conditions reduces to

$$\frac{d^2T}{dx^2}+\frac{q_g}{k}=0 \qquad (1)$$

Integrating equation (1) with respect to ‘x’, we get

$$\frac{dT}{dx}+\frac{q_g}{k}x=C_1 \tag{2}$$

Integrating equation (2) again with respect to ‘x’, we get

$$T+\frac{q_g}{k}\frac{x^2}{2}=C_1x+C_2 \tag{3}$$

Using the boundary conditions,

At x = 0, $\frac{dT}{dx}=0$, From equation (2), we get

$$C_1=0 \tag{4}$$

At x = L/2, T = T_1, From equation (3), we get

$$T_1+\frac{q_g}{k}\frac{L^2}{8}=C_1\frac{L}{2}+C_2$$

As $C_1=0$, we can write

$$T_1+\frac{q_g}{k}\frac{L^2}{8}=C_2 \tag{5}$$

Substituting the values of C_1 and C_2 in equation (3), we get

$$T+\frac{q_g}{k}\frac{x^2}{2}=T_1+\frac{q_g}{k}\frac{L^2}{8}$$

$$T=T_1+\frac{q_g}{k}\frac{L^2}{8}\left(1-\left(\frac{2x}{L}\right)^2\right) \tag{6}$$

Equation (6) represents temperature distribution equation in the slab having a heat generating source present inside it.

Temperature will be maximum at the center of the slab where x = 0

$$T_{\max}=T_1+\frac{q_g}{k}\frac{L^2}{8} \tag{7}$$

Flow of heat can be expressed as:

$$Q=-kA\left(\frac{dT}{dx}\right)_{x=L/2} \tag{8}$$

Using equation (2), we can write

$$\left(\frac{dT}{dx}\right)_{x=L/2} = -\frac{q_g}{k}\frac{L}{2} \qquad (9)$$

Substituting value of $\left(\frac{dT}{dx}\right)_{x=L/2}$ from equation (9) in equation (8), we get

$$Q = -kA\left(-\frac{q_g}{k}\frac{L}{2}\right)$$

$$Q = AL\frac{q_g}{2} \qquad (10)$$

Equation (10) represents flow from one of sides of the slab; therefore, total heat flow from both the sides is expressed as

$$Q_{Total} = 2\times AL\frac{q_g}{2} = ALq_g$$

Total Heat Conducted from both sides of the slab = Volume x Heat generating capacity

Total Heat Conducted from both sides of the slab = Total Heat generated

Under steady state conditions, heat conducted at x = L/2 must be equal to convected from a side to the atmospheric air. Therefore,

$$Q = AL\frac{q_g}{2} = hA\left(T_1 - T_a\right)$$

$$T_1 = T_a + \frac{q_g L}{2h} \qquad (11)$$

Substituting the value of T_1 from equation (11) in equation (6), we get

$$T = T_a + \frac{q_g L}{2h} + \frac{q_g}{k}\frac{L^2}{8}\left(1-\left(\frac{2x}{L}\right)^2\right) \qquad (12)$$

Equation (12) represents temperature distribution, if one side of the slab is insulated

At one side, temperature distribution will be represented by equation (6) except that L/2 will be replaced by L and is expressed as

$$T = T_1 + \frac{q_g}{k}\frac{L^2}{8}\left(1-\left(\frac{2x}{2L}\right)^2\right)$$

$$T = T_1 + \frac{q_g}{k}\frac{L^2}{8}\left(1 - \left(\frac{x}{L}\right)^2\right)$$

ii) When Temperature of Both Sides of Slab are Different:

If the heat source present inside the slab generates heat q_g per unit volume and heat distribution in towards both slabs is not uniform then the temperature of both sides of the slab will be different as shown in Figure 2.

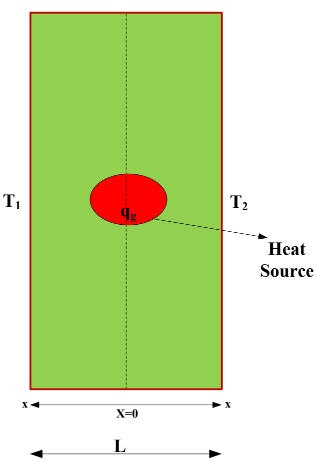

Figure 2

The differential equation governing the heat flow through the slab is expressed as:

$$\frac{d^2T}{dx^2} + \frac{q_g}{k} = 0 \qquad (13)$$

Integrating equation (13) with respect to ‘x’, we get

$$\frac{dT}{dx} + \frac{q_g}{k}x = C_1 \qquad (14)$$

Integrating equation (4.14) again with respect to ‘x’, we get

$$T + \frac{q_g}{k}\frac{x^2}{2} = C_1 x + C_2 \qquad (15)$$

Applying the boundary conditions,

At x = 0, $T = T_1$, From equation (15), we get

$$C_2 = T_1 \qquad (16)$$

At x = L, $T = T_2$, From equation (15), we get

$$\frac{T_2 - T_1}{L} + \frac{q_g L}{2k} = C_1 \tag{17}$$

Substituting the values of C_1 and C_2 in equation (15), we get

$$T + \frac{q_g}{k}\frac{x^2}{2} = x\left[\frac{T_2 - T_1}{L} + \frac{q_g L}{2k}\right] + T_1$$

$$T = T_1 + \frac{q_g L x}{2k} - \frac{q_g x^2}{2k} + \frac{x}{L}\left(T_2 - T_1\right) \tag{18}$$

Subtracting T_2 from both sides of the equation (18), we get

$$T - T_2 = T_1 - T_2 + \frac{q_g L x}{2k} - \frac{q_g x^2}{2k} + \frac{x}{L}\left(T_2 - T_1\right)$$

$$T - T_2 = T_1 - T_2 + \frac{q_g}{2k}(Lx - x^2) + \frac{x}{L}\left(T_2 - T_1\right)$$

$$T - T_2 = T_1 - T_2 + \frac{q_g}{2k}(Lx - x^2) + \frac{x}{L}\left(T_2 - T_1\right) \tag{19}$$

Dividing both sides of the equation (19) with T_1-T_2, we get

$$\frac{T - T_2}{T_1 - T_2} = 1 + \frac{q_g}{2k(T_1 - T_2)}(Lx - x^2) + \frac{x}{L}\frac{\left(T_2 - T_1\right)}{\left(T_1 - T_2\right)}$$

$$\frac{T - T_2}{T_1 - T_2} = 1 + \frac{q_g}{2k(T_1 - T_2)}(Lx - x^2) - \frac{x}{L}\frac{\left(T_1 - T_2\right)}{\left(T_1 - T_2\right)}$$

$$\frac{T - T_2}{T_1 - T_2} = 1 - \frac{x}{L} + \frac{q_g}{2k(T_1 - T_2)}(Lx - x^2)$$

$$\frac{T - T_2}{T_1 - T_2} = 1 - \frac{x}{L} + \frac{q_g}{2k(T_1 - T_2)}(Lx - x^2)\frac{L^2}{L^2}$$

$$\frac{T - T_2}{T_1 - T_2} = 1 - \frac{x}{L} + \frac{q_g L^2}{2k(T_1 - T_2)}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right]$$

$$\frac{T - T_2}{T_1 - T_2} = \left(1 - \frac{x}{L}\right) + \frac{q_g L^2}{2k(T_1 - T_2)}\frac{x}{L}\left(1 - \frac{x}{L}\right)$$

$$\frac{T - T_2}{T_1 - T_2} = \left(1 - \frac{x}{L}\right)\left[1 + \frac{q_g L^2}{2k(T_1 - T_2)} \frac{x}{L}\right]$$

$$\frac{T - T_2}{T_1 - T_2} = \left(1 - \frac{x}{L}\right)\left(1 + \frac{B\,x}{L}\right) \qquad (20)$$

Where $B = \dfrac{q_g L^2}{2k(T_1 - T_2)}$

Equation (20) represents temperature distribution equation in the slab having a heat generating source present inside it. In order to find out the location of maximum temperature in the slab equation (20) is differentiated with respect to 'x' and equated equal to zero.

$$\frac{d}{dx}\left(\frac{T - T_2}{T_1 - T_2}\right) = \frac{d}{dx}\left[\left(1 - \frac{x}{L}\right)\left(1 + \frac{B\,x}{L}\right)\right]$$

$$\frac{dT}{dx} = \frac{d}{dx}\left[\left(1 + \frac{B\,x}{L} - \frac{x}{L} - \frac{B\,x^2}{L^2}\right)\right]$$

$$0 = \left[\left(\frac{B}{L} - \frac{1}{L} - \frac{2B\,x}{L^2}\right)\right]$$

$$0 = \frac{1}{L}\left(B - 1 - \frac{2B\,x}{L}\right)$$

$$0 = \left(B - 1 - \frac{2B\,x}{L}\right)$$

$$\frac{2B\,x}{L} = (B - 1)$$

$$\frac{x}{L} = \frac{B - 1}{2B} \qquad (21)$$

Equation (21) gives the location of maximum temperature in the slab. The equation representing the maximum value of temperature is obtained by substituting the value of maximum x /L from equation (21) into equation (20).

$$\frac{T_{max} - T_2}{T_1 - T_2} = \left(1 - \frac{B - 1}{2B}\right)\left(1 + \frac{B\,(B - 1)}{2B}\right)$$

$$\frac{T_{\max}-T_2}{T_1-T_2}=\left(\frac{2B-B+1}{2B}\right)\left(\frac{2+B-1}{2}\right)$$

$$\frac{T_{\max}-T_2}{T_1-T_2}=\frac{(B+1)^2}{4B} \qquad (22)$$

Flow of heat from one surface is given as

$$Q_1=-kA\left(\frac{dT}{dx}\right)_{x=0}$$

From equation (14) substituting the value of dT/dx, we get

$$Q_1=-kA\left(C_1-\frac{q_g}{k}x\right)_{x=0}$$

Substituting the value of C_1 from equation (17), we get

$$Q_1=-kA\left(\frac{T_2-T_1}{L}+\frac{q_g}{2k}L-\frac{q_g}{k}x\right)_{x=0}$$

$$Q_1=kA\left(\frac{T_1-T_2}{L}-\frac{q_g}{2k}L\right)$$

Similarly heat flow from the other surface

$$Q_2=-kA\left(\frac{dT}{dx}\right)_{x=L}$$

$$Q_2=-kA\left(\frac{T_2-T_1}{L}+\frac{q_g}{2k}L-\frac{q_g}{k}x\right)_{x=L}$$

$$Q_2=-kA\left(\frac{T_2-T_1}{L}+\frac{q_g}{2k}L-\frac{q_g}{k}L\right)$$

$$Q_2=kA\left(\frac{T_1-T_2}{L}-\frac{q_g}{2k}L+\frac{q_g}{k}L\right)$$

$$Q_2=kA\left(\frac{T_1-T_2}{L}+\frac{q_g}{2k}L\right)$$

In case maximum temperature occurs inside the slab, heat will flow from both surfaces of the slab and total heat flow will be given as:

$Q_T = Q_1 + Q_2$

In case T_1 is the maximum temperature, heat will flow towards x (+ve only) and heat lost will be given as:

$Q_T = Q_2$

One-Dimensional Heat Flow through a cylinder with Heat Generation

i) A hollow Cylinder:

Consider a hollow cylinder of length L having inner and outer radii r_1 and r_2 respectively in which flow of heat is unidirectional along the radial direction. T_1 and T_2 are temperatures of the inner and outer surfaces of the cylinder respectively. In order to determine temperature distribution and heat flow rate, a small element at radius r and thickness dr is considered. A heat source present inside this elemental strip is generating q_g amount of heat per unit volume as shown in Figure 3.

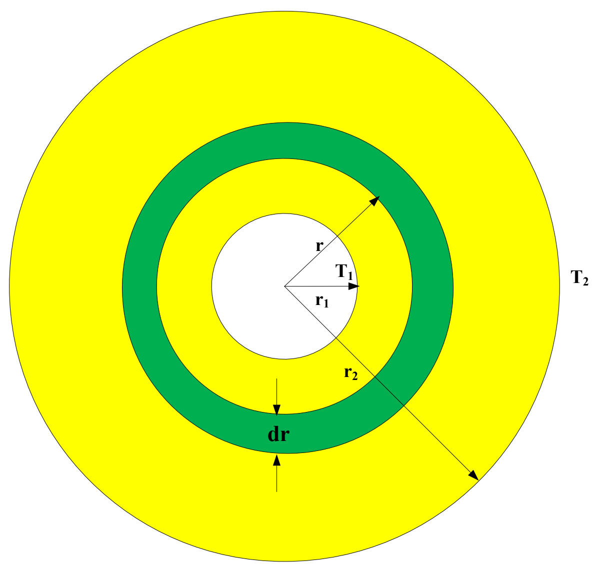

Figure 3

Heat conducted into the element, $Q_r = -k(2\,\pi\,r\,L)\,dT/dr$ (23)

Heat generated in the element, $Q_g = 2\,\pi\,r\,L\,dr\,q_g$ (24)

Heat conducted out of the element, $Q_{r+dr} = Q_r + \frac{d}{dr}(Q_r)\,dr$ (25)

For steady state condition of heat flow

Heat conducted into the element + Heat generated in the element = Heat conducted out of the element

$$Q_r + Q_g = Q_{r+dr}$$

$$Q_r + Q_g = Q_r + \frac{d}{dr}(Q_r)\,dr$$

$$Q_g = \frac{d}{dr}(Q_r)\,dr \tag{26}$$

Substituting the values of Q_r and Q_g from equations (23 and 24) in equation (26), we get

$$2\pi rLdrq_g = \frac{d}{dr}\left(-2\pi rLk\frac{dT}{dr}\right)dr$$

$$rq_g = -k\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{rq_g}{k} = 0 \tag{27}$$

In order to find out the solution of the above equation, integrate it with respect to r

$$r\frac{dT}{dr} + \frac{r^2 q_g}{2k} = C_1 \tag{28}$$

$$\frac{dT}{dr} + \frac{rq_g}{2k} = \frac{C_1}{r} \tag{29}$$

Integrating equation (28) again with respect to r, we get

$$T + \frac{q_g}{4k}r^2 = C_1 \log_e r + C_2 \tag{30}$$

C_1 and C_2 are constants of integration and the expressions for these constants can be found out by using the following boundary conditions

At $r=r_1$, $T=T_1$ and at $r=r_2$, $T=T_2$

$$T_1 + \frac{q_g}{4k}r_1^2 = C_1 \log_e r_1 + C_2 \tag{31}$$

$$T_2 + \frac{q_g}{4k}r_2^2 = C_1 \log_e r_2 + C_2 \tag{32}$$

Subtracting equation (32) from equation (31), we get

$$T_1 - T_2 + \frac{q_g}{4k}\left(r_1^2 - r_2^2\right) = C_1 \log_e \frac{r_1}{r_2}$$

$$C_1 \log_e \frac{r_2}{r_1} = \frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)$$

$$C_1 = \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}} \tag{33}$$

Substituting the value of C_1 in equation (31), we get

$$T_1 + \frac{q_g}{4k} r_1^2 = \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}} \log_e r_1 + C_2$$

$$C_2 = T_1 + \frac{q_g}{4k} r_1^2 - \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}} \log_e r_1 \tag{34}$$

Substituting the values of C_1 and C_2 in equation (30), we get

$$T + \frac{q_g}{4k} r^2 = \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}} \log_e r + T_1 + \frac{q_g}{4k} r_1^2 - \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}} \log_e r_1$$

$$T + \frac{q_g}{4k} r^2 = T_1 + \frac{q_g}{4k} r_1^2 + \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}} \left(\log_e r - \log_e r_1\right)$$

$$T - T_1 = \frac{q_g}{4k}\left(r_1^2 - r^2\right) + \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}} \left(\log_e \frac{r}{r_1}\right) \tag{35}$$

Dividing both sides of equation (35) by T_1-T_2, we get

$$\frac{T - T_1}{T_1 - T_2} = \frac{q_g}{4k\left(T_1 - T_2\right)}\left(r_1^2 - r^2\right) + \frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right) - \left(T_1 - T_2\right)}{\left(T_1 - T_2\right)\log_e \frac{r_2}{r_1}} \left(\log_e \frac{r}{r_1}\right)$$

$$\frac{T - T_1}{T_1 - T_2} = \frac{q_g\left(r_1^2 - r^2\right)}{4k\left(T_1 - T_2\right)} + \left(\frac{\frac{q_g}{4k}\left(r_2^2 - r_1^2\right)}{\left(T_1 - T_2\right)} - 1\right) \frac{1}{\log_e \frac{r_2}{r_1}} \left(\log_e \frac{r}{r_1}\right)$$

$$\frac{T-T_1}{T_1-T_2}=\frac{q_g\left(r_1^2-r^2\right)}{4k\left(T_1-T_2\right)}+\frac{\frac{q_g}{4k}\left(r_2^2-r_1^2\right)}{\left(T_1-T_2\right)}\frac{\log_e\frac{r}{r_1}}{\log_e\frac{r_2}{r_1}}-\frac{\log_e\frac{r}{r_1}}{\log_e\frac{r_2}{r_1}}$$

$$\frac{T-T_1}{T_1-T_2}=\frac{q_g}{4k\left(T_1-T_2\right)}\left[\left(r_1^2-r^2\right)+\left(r_2^2-r_1^2\right)\frac{\log_e\frac{r}{r_1}}{\log_e\frac{r_2}{r_1}}\right]-\frac{\log_e\frac{r}{r_1}}{\log_e\frac{r_2}{r_1}}$$

Multiplying and dividing Right Hand Side of the above equation by r^2, we get

$$\frac{T-T_1}{T_1-T_2}=\frac{q_g\,r^2}{4k\left(T_1-T_2\right)}\left[\left(\frac{r_1^2}{r^2}-\frac{r^2}{r^2}\right)+\left(\frac{r_2^2}{r^2}-\frac{r_1^2}{r^2}\right)\frac{\log_e\frac{r}{r_1}}{\log_e\frac{r_2}{r_1}}\right]-\frac{\log_e\frac{r}{r_1}}{\log_e\frac{r_2}{r_1}} \qquad (36)$$

Equation (36) represents temperature distribution inside a hollow cylinder with heat generation.

ii) A Solid Cylinder

In case of solid cylinder, the governing equation remains same as equation (30)

$$T+\frac{q_g}{4k}r^2=C_1\log_e r+C_2$$

Differentiating above equation with respect to r, we get

$$\frac{dT}{dr}+\frac{rq_g}{2k}=\frac{C_1}{r}$$

Applying the boundary conditions

At r=0, $\frac{dT}{dr}=0$, so $C_1=0$

At $r=r_2$, $T=T_2$,

$$C_2=T_2+\frac{q_g}{4k}r_2^2$$

Substituting the values of C_1 and C_2 in equation (30), we get

$$T+\frac{q_g}{4k}r^2=T_2+\frac{q_g}{4k}r_2^2$$

$$T=T_2+\frac{q_g r_2^2}{4k}\left[1-\left(\frac{r}{r_2}\right)^2\right]$$

$$T - T_2 = \frac{q_g r_2^2}{4k}\left[1 - \left(\frac{r}{r_2}\right)^2\right] \qquad (37)$$

Equation (37) represents temperature distribution equation in a solid cylinder with heat generation. Maximum temperature will occur at center of the cylinder where r=0, and will be expressed as

$$T_{\max} = T_2 + \frac{q_g r_2^2}{4k}$$

$$T_{\max} - T_2 = \frac{q_g r_2^2}{4k} \qquad (38)$$

Dividing equation (37) by equation (38), we get

$$\frac{T - T_2}{T_{\max} - T_2} = \left[1 - \left(\frac{r}{r_2}\right)^2\right] \qquad (39)$$

Heat flow through a solid cylinder is expressed as

$$Q = \left[-kA\frac{dT}{dr}\right]_{r=r_2}$$

$$Q = -k2\pi r_2 L\frac{d}{dr}\left[T_2 + \frac{q_g r_2^2}{4k}\left\{1 - \left(\frac{r}{r_2}\right)^2\right\}\right]_{r=r_2}$$

$$Q = -k2\pi r_2 L\frac{d}{dr}\left[\frac{q_g}{4k}\{-2r\}\right]_{r=r_2}$$

$$Q = \frac{4\pi k r_2^2 L q_g}{4k}$$

$$Q = \pi r_2^2 L q_g \qquad (40)$$

Heat conducted = Volume of cylinder x heat generating capacity per unit volume per unit Time

For steady state conditions, heat conducted at r = r_2 must be equal to heat convected from outer surface of cylinder to the surrounding fluid.

Heat Conducted = Heat convected

From equation (40), we can write

$$\pi r_2^2 L q_g = h \times 2\pi r_2 L\left(T_2 - T_f\right)$$

T_f is temperature of fluid surrounding the cylinder.

$$T_2 = T_f + \frac{r_2 q_g}{2h}$$

Substituting the value of T_2 in equation (37), we get

$$T = T_f + \frac{r_2 q_g}{2h} + \frac{r_2^2 q_g}{4k}\left[1 - \left(\frac{r}{r_2}\right)^2\right]$$

One-Dimensional Heat Flow through a sphere with Heat Generation

Consider steady state heat conduction through a hollow sphere having r_1 and r_2 as inner and outer radii respectively. Temperature of the inner and outer surfaces is T_1 and T_2 respectively. Heat is flowing from inner to outer surface as T_1 is greater than T_2 as shown in Figure 4.

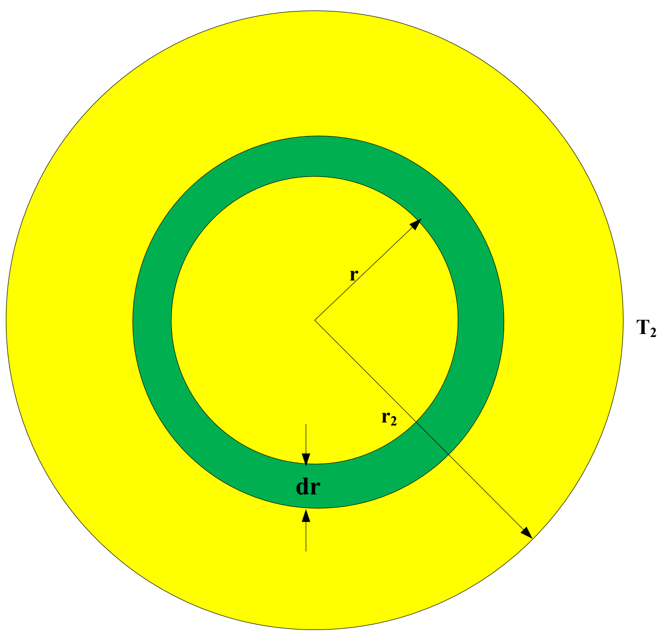

Figure 4

The general conduction equation which governs the conduction heat transfer is written as

$$\left[\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 T}{\partial \phi^2}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{q_g}{k}\right]=\frac{1}{\alpha}\ \frac{\partial T}{\partial t}dt \tag{41}$$

Since it is a case of one-dimensional, stead heat conduction through a wall of uniform conductivity with heat generation, therefore, $\frac{dT}{dt}=0$ and $\frac{dT}{d\phi}=\frac{dT}{d\theta}=0$

Therefore, equation (41) reduces to

$$\frac{d^2T}{dr^2}+\frac{2}{r}\frac{dT}{dr}+\frac{q_g}{k}=0 \tag{42}$$

The above equation can be written as

$$r\frac{d^2T}{dr^2}+\frac{dT}{dr}+\frac{dT}{dr}+\frac{q_g}{k}r=0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{dT}{dr}+\frac{q_g}{k}r=0 \tag{43}$$

Integrating equation (43) with respect to r, we get

$$r\frac{dT}{dr}+T+\frac{q_g}{k}\frac{r^2}{2}=C_1$$

$$\frac{d}{dr}(rT)+\frac{q_g}{k}\frac{r^2}{2}=C_1$$

Upon integrating above equation once more with respect to r, we get

$$rT+\frac{q_g}{k}\frac{r^3}{6}=C_1r+C_2 \tag{44}$$

Applying the first boundary condition i.e. at $r = 0$, $dT/dr = 0$ to equation (44), we get

$$C_2 = 0 \tag{45}$$

Applying the second boundary condition i.e at $r = r_2$, $T = T_2$ to equation (43), we get

$$r_2T_2+\frac{q_g}{k}\frac{r_2^3}{6}=C_1r_2$$

$$T_2 + \frac{q_g}{k}\frac{r_2^2}{6} = C_1 \tag{46}$$

Substituting the values of C_1 and C_2 in equation (44), we get

$$r\,T + \frac{q_g}{k}\frac{r^3}{6} = r\left(T_2 + \frac{q_g}{k}\frac{r_2^2}{6}\right)$$

$$T + \frac{q_g}{k}\frac{r^2}{6} = \left(T_2 + \frac{q_g}{k}\frac{r_2^2}{6}\right)$$

$$T = T_2 + \frac{q_g}{k}\frac{r_2^2}{6}\left(1 - \left(\frac{r}{r_2}\right)^2\right) \tag{47}$$

Equation (47) represents temperature distribution equation in a solid sphere having a heat source present inside it.

Heat flow rate through a sphere with heat generation can be determined by using the following equation

$$Q = -kA\left(\frac{dT}{dr}\right)_{r=r_2}$$

$$Q = -k4\pi r_2^2 \frac{d}{dr}\left(T_2 + \frac{q_g}{6k}\left(r_2^2 - r^2\right)\right)_{r=r_2}$$

$$Q = -4\pi k r_2^2 \left(\frac{q_g}{6k}(-2r)\right)_{r=r_2}$$

$$Q = 4\pi k r_2^2 \frac{q_g}{3k} r_2$$

$$Q = \frac{4}{3}\pi r_2^3\, q_g$$

Heat conducted = Volume of sphere x heat generating capacity

For steady state conditions, heat conducted through a sphere must be equal to heat convected from outer surface of the sphere

$$\frac{4}{3}\pi r_2^3 q_g = 4\pi r_2^2 h(T_2 - T_f)$$

$$T_2 = T_f + \frac{q_g r_2}{3h}$$

Substitute the value of T_2 from above equation in equation (47), we get

$$T = T_2 + \frac{q_g r_2}{3h} + \frac{q_g r_2^2}{6k}\left(1 - \left(\frac{r}{r_2}\right)^2\right)$$

REVIEW QUESTIONS:

Q.1 A solid cement wall of a building having thermal conductivity k and thickness δ is heated by convection on the inner side and cooled by convection on the outside. The heat flux through the wall can be expressed as

a) $\frac{(T_1 - T_2)}{1/h_1 + \delta/k + 1/h_2}$

b) $\frac{(T_1 - T_2)}{h/_1 + \delta/_k + h/_2}$

c) $\frac{k(T_1 - T_2)(h_1 + h_2)}{\delta}$

d) $\frac{(T_1 - T_2)}{1/h_1 + k/\delta + 1/h_2}$

Q.2 Heat is transferred from a hot fluid to a cold fluid through a plane wall of thickness δ, surface area A and thermal conductivity k. The thermal resistance for the setup is

a) $\frac{1}{A}\left(\frac{1}{h_1} + \frac{\delta}{k} + \frac{1}{h_2}\right)$

b) $A\left(\frac{1}{h_1} + \frac{\delta}{k} + \frac{1}{h_2}\right)$

c) $\frac{1}{A}\left(h_1 + \frac{k}{\delta} + h_2\right)$

d) $A\left(h_1 + \frac{k}{\delta} + h_2\right)$

Q.3 A gas turbine blade (idealized as a flat plate of surface area A, thickness δ and thermal conductivity k) has hot gases at temperature T_1 on one side and cooling air at temperature T_2 on the other side. If h_1 and h_2 are the corresponding surface coefficients of heat transfer, then the overall heat transfer coefficient U is given by

a) $\frac{1}{U} = \frac{1}{h_1} + \frac{\delta}{k} + \frac{1}{h_2}$

b) $\frac{1}{U} = \frac{1}{h_1} + \frac{k}{\delta} + \frac{1}{h_2}$

c) $U = h_1 + \frac{\delta}{k} + h_2$

d) $U_1 = \frac{1}{h_1} + \frac{\delta}{k} + \frac{1}{h_2}$

Q.4 Choose the false statement

a) The unit of heat transfer coefficient is kcal/m_2-hr- °C

b) The overall heat transfer coefficient has units of W/m^2-deg

c) In M-L-T-θ system, the dimensions of convective heat transfer coefficient and overall heat transfer coefficient are $MT^{-3}\theta^{-1}$

d) the overall heat transfer coefficient is the resistance to heat flow

Q.5 A hollow cylinder of inner radius r_1 and outer radius r_2 is subjected to steady state heat transfer which results in constant surface temperature t_1 and t_2 at radii r_1 and r_2 respectively.

For constant thermal conductivity k, the radial heat flow per unit length of cylinder is given by

a) $\frac{2\pi k(T_1-T_2)}{\log_e(r_2/r_1)}$

b) $\frac{2\pi(T_1-T_2)}{k\log_e(r_2/r_1)}$

c) $\frac{(T_1-T_2)}{2\pi k\log_e(r_2/r_1)}$

d) $\frac{k(T_1-T_2)}{2\pi\log_e(r_2/r_1)}$

Q.6 A cylindrical pipe of length l has inner radius r_1 and outer radius r_2. The interior of pipe carries hot water at temperature t1 whereas outer surface of the pipe is at temperature t2 ($T_2 > T_1$). The rate of conduction heat loss per unit length of the pipe is gives as

a) $\frac{4\pi k(T_1-T_2)}{(r_2/r_1)}$

b) $\frac{2\pi k\ (T_1-T_2)}{(r_2/r_1)}$

c) $\frac{2\pi k(T_1-T_2)}{\log_e(r_2/r_1)}$

d) $4\pi k(T_1 - T_2)(r_1 + r_2)$

Q.7 For steady state and constant value of thermal conductivity, the temperature distribution associated with radial conduction through a cylinder has a

a) linear

b) logarithmic

c) parabolic

d) exponential curve

Q.8 The heat flow equation through a cylinder of inner radius r_1 and outer radius r_2 is desired to be written in the same form as that for heat flow through a plane wall. For wall thickness ($r_1 – r_2$), the equivalent area A_m would be

a) $\frac{A_1+A_2}{2}$

b) $\frac{A_1+A_2}{2\log_e(A_2/A_1)}$

c) $\frac{A_2-A_1}{\log_e(A_2/A_1)}$

d) $\frac{A_2+A_1}{2\log_e(A_2/A_1)}$