LESSON 5

Electrical Analogy For Conduction Problems

Consider heat flowing through a slab of thickness ‘L’ and area “A’ and T_1 and T_2 are the temperatures on the two faces of the slab as shown in Figure 1. Heat transfer from high temperature side to low temperature side is expressed as

$$Q = \frac{k\,A}{L}\left(T_1 - T_2\right) \qquad (1)$$

Where $(T_1 - T_2)$ is the thermal potential

$\frac{L}{k\,A}$ is the thermal resistance

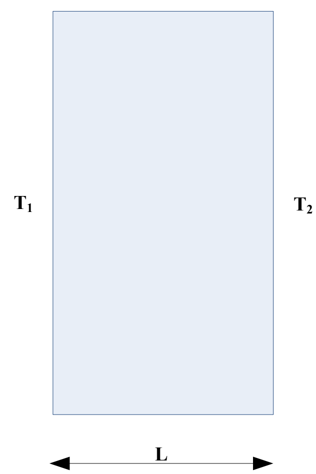

Figure 1

Now, consider an electric circuit having resistance ‘R’ and electric potential E_1 and E_2 at the ends as shown in Figure 2. Current ‘I’ passing through the circuit can be expressed as

$$I = \frac{\left(E_1 - E_2\right)}{R} \qquad (2)$$

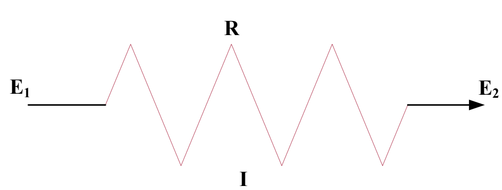

Figure 2

Equations (1) and (2) are found to be symmetrical on comparison. 'Q' amount of heat flows through the slab having thermal resistance $\frac{L}{k\,A}$ when a thermal potential ($T_1 - T_2$) exits. Similarly, 'I' amount of current passes through the circuit having resistance 'R' when an electric potential ($E_1 - E_2$) exists. Therefore, flow of heat through the slab can be represented by an electric circuit as shown in Figure 3.

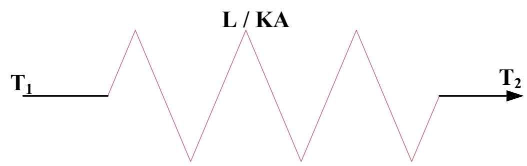

Figure 3

If a hot gas at temperature T_g is in contact with one side of the slab and air at temperature T_a at the other side, then heat transfer from the hot gas to air through this slab of thickness can be represented by an electric circuit as shown in Figure 4

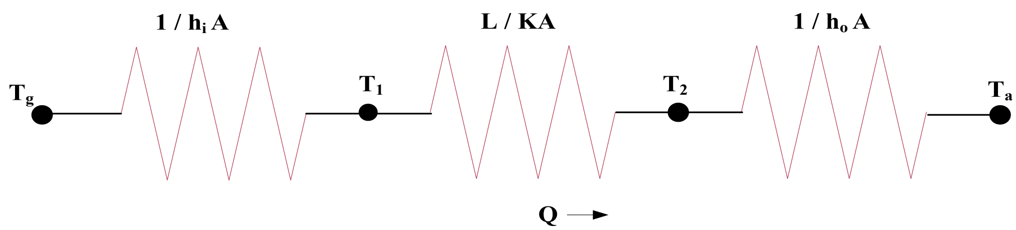

Figure 4

Heat transfer from hot gas to air can be expressed as

$$Q = \frac{(T_g - T_a)}{1/h_i A + L/KA + 1/h_0 A} \tag{3}$$

For a slab made of three material having thermal conductivities K_A, K_B and K_C respectively and is exposed to a hot gas on one side and atmospheric air on the other side as shown in Figure 5, an equivalent electric circuit has been shown in Figure 6. Heat transfer from the hot gas to atmospheric air is expressed as

$$Q = \frac{(T_g - T_a)}{1/h_i A + L_1/K_A A_A + L_2/K_B A_B + L_3/K_C A_C + 1/h_0 A} \tag{4}$$

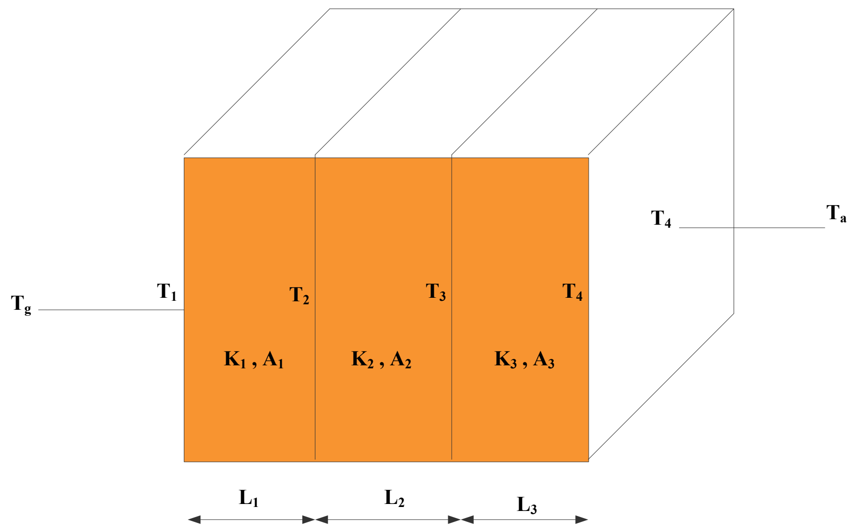

Figure 5

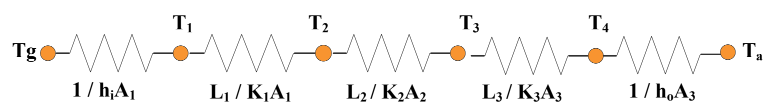

Figure 6

Similarly for the composite slab shown in Figure 7, an equivalent electric circuit has been shown in Figure 8.

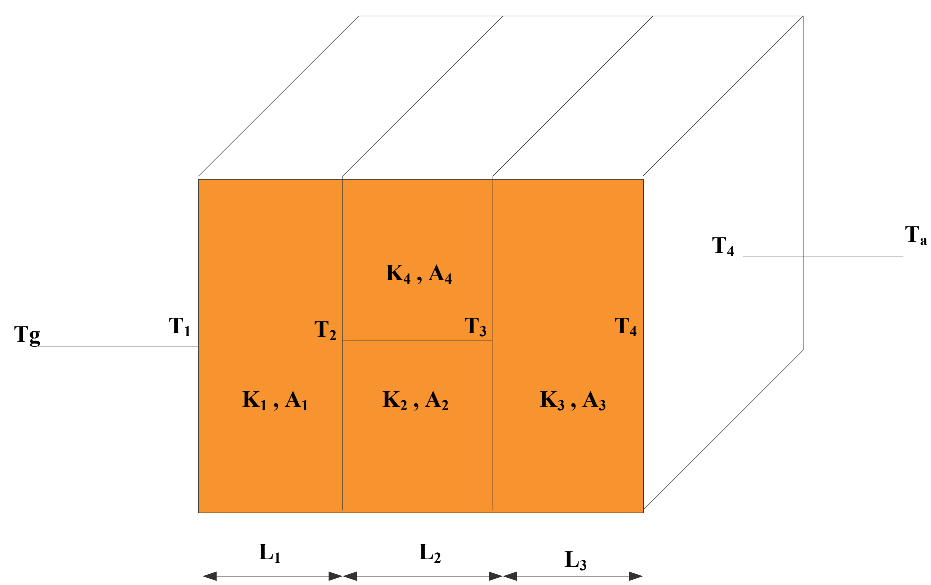

Figure 7

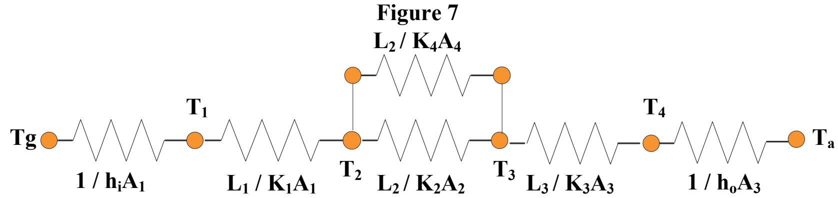

Figure 8

SOLVED EXAMPLES ON CONDUCTION

Ex. 5.1 The inner and outer surfaces of a wall of a room are maintained at 25°C and -20°C respectively. The wall comprises of following three different materials:

2.5 cm inner mica sheet having thermal conductivity, k_1 =0.60 W/m-deg

10 cm intermediate layer of an insulating material having thermal conductivity, k_2 =1.90 W/m-deg

15 cm outer brick work having thermal conductivity, k_3 = 0.85 W/m-deg

Determine the rate of heat loss per unit area of the wall and interface temperatures of the wall.

Solution:

Given: T_1= 25°C, T_4= -20°C, Thickness of mica sheet, δ_1=2.5 cm = 0.025 m
Thickness of insulating material, δ_2=10 cm = 0.10 m
Thickness of brick work, δ_3=15 cm = 0.15 m
Thermal conductivity of mica sheet, k_1 = 0.60 W/m-deg
Thermal conductivity of insulating material, k_2 = 1.90 W/m-deg
Thermal conductivity of brick work, k_3 = 0.85 W/m-deg
$A_1 = A_2 = A_3 = 1\ m^2$

To determine: i) Rate of heat loss per unit area, Q

$$= \frac{\Delta T}{\sum R_t}$$

$$\Delta T = T_1 - T_4 = 25 - (-20) = 45°C$$

$$\sum R_t = R_{t_1} + R_{t_2} + R_{t_3}$$

$$R_{t_1} = \frac{\delta_1}{k_1 A_1} = \frac{0.025}{0.60 \times 1} = 0.042\ deg/W$$

$$R_{t_2} = \frac{\delta_2}{k_2 A_2} = \frac{0.1}{1.90 \times 1} = 0.053 deg/W$$

$$R_{t_3} = \frac{\delta_3}{k_3 A_3} = \frac{0.15}{0.85 \times 1} = 0.177\ deg/W$$

Total resistance $R_t = \sum R_t$ = 0.042+0.053+0.177= **0.272 deg/W**

$\therefore$ Heat loss $Q = \frac{\Delta T}{\sum R_t} = \frac{25-(-20)}{0.272} = 165.44\ W$

ii) Interface temperatures, T_2 and T_3

$$T_2 = T_1 - Q\,R_{t_1} = 25 - (165.44 X 0.042) = 18.05\ °C$$

$$T_3 = T_2 - Q\,R_{t_2} = 18.05 - (165.44 X 0.053) = 9.28\ °C$$

Ex 5.2 A furnace wall is a made up of fire bricks 80 mm thick (k = 52.8 kJ/m-hr-deg) lined on inside with metallic lining 100 mm thick (k = 21.32 kJ/m-hr-deg) and on the outside with red bricks 250 mm thick (k = 18.84 kJ/m-hr-deg). The inside and outside surfaces of the wall are at temperature 875° C and 25 °C respectively. Determine the heat loss from unit area of the wall. It is desired that the heat loss be reduced to 10 MJ/hour by replacing mettalic linin with an air gap between fire bricks and red bricks. Estimate the necessary width of air gap if thermal conductivity for air is 0.126 kJ/m-hr-deg.

Solution:

Given: T_1= 875°C, T_4= 25°C, Thickness of fire bricks, δ_1=80 mm = 0.08 m
Thickness of mettalic lining, δ_2=100 mm = 0.10 m
Thickness of red brick, δ_3=250 mm = 0.25 m
Thermal conductivity of fire bricks, k_1 = 52.8 kJ/m-hr-deg
Thermal conductivity of mettalic lining, k_2 = 21.32 kJ/m-hr-deg
Thermal conductivity of red brick, k_3 = 18.84 kJ/m-hr-deg
$A_1 = A_2 = A_3 = 1\ m^2$

To determine: i) Rate of heat loss per unit area, Q

$$= \frac{\Delta T}{\sum R_t}$$

$$\Delta T = T_1 - T_4 = 875 - 25 = 850°C$$

$$\sum R_t = R_{t_1} + R_{t_2} + R_{t_3}$$

$$R_{t_1} = \frac{\delta_1}{k_1 A_1} = \frac{0.08}{52.8 \times 1} = 0.00151\ deg - hr/kJ$$

$$R_{t_2} = \frac{\delta_2}{k_2 A_2} = \frac{0.1}{21.32 \times 1} = 0.0047\ deg - hr/kJ$$

$$R_{t_3} = \frac{\delta_3}{k_3 A_3} = \frac{0.25}{18.84 \times 1} = 0.013\ deg - hr/kJ$$

Total resistance, $R_t = 0.00151 + 0.0047 + 0.013 = 0.01921\ deg - hr/kJ$

$$Heat\ loss, Q = \frac{\Delta T}{\sum R_t} = \frac{875 - 25}{0.01921} = 44247.78 \frac{kJ}{hr} = 44.247\ MJ/hr$$

In order to reduce the heat loss to 10 MJ/hr, total thermal resistance, $\sum R_t$ should be

$$\sum R_t = \frac{\Delta T}{Q} = \frac{875 - 25}{10\ X\ 1000} = 0.08578\ deg - hr/kJ$$

Now, total thermal resistance is the sum of thermal resistances offered by fire bricks, air gap and red bricks.

∴ Thermal resistance for the air gap = Total Thermal Resistance – Sum of thermal resistances ofered by fire and red bricks

$$\frac{\delta}{kA} = 0.0878 - (\mathbf{0.00151} + \mathbf{0.013}) = \mathbf{0.07329\ deg\ \boldsymbol{hr/kJ}}$$

∴ Thickness of air gap, $\delta = 0.07329 \times 0.126 =$ **9.23 mm**