LESSON-6

SOLVED EXAMPLES ON CONDUCTION

Ex 6.1 Find the heat flow rate per unit area through a composite wall made of four different materials as shown in Figure 1. Thermal conductivites are as:

K_1 = 50 W/m-deg; k_2 = 80 W/m-deg; k_3 = 150 W/m-deg; k_4 = 55 W/m-deg

L_1 = 25 cm, L_2 = 85 cm, L_3 = 40 cm., L_4 = 85 cm.,

Temperatures on left face and right face are 525 °C and 50 °C respectively.

Solution: The arrangement of the composite system and its equivalent thermal circuit for heat flow has been shown in Figures 2 and 3.

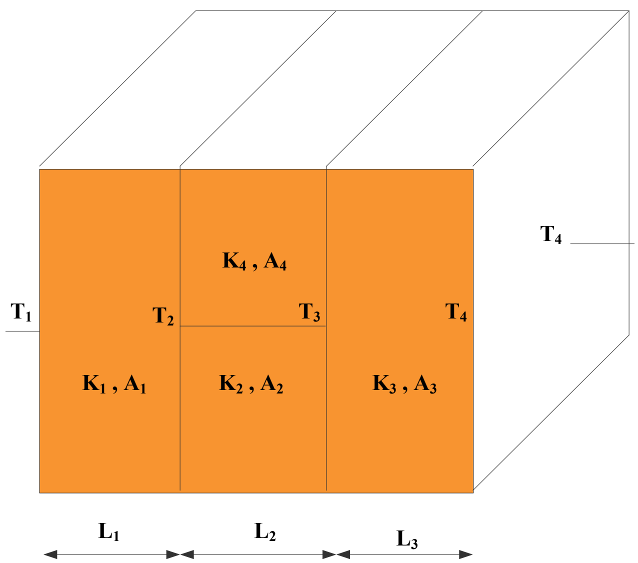

Figure 1

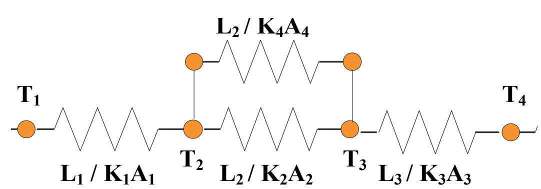

Figure 2

Given: T_1= 525°C, T_4= 50°C,

Thickness of Ist material, L_1=25 cm = 0.25 m, Thickness of IInd material, L_2 = 85 cm = 0.85 m

Thickness of IIIrd material, L_3= 40 cm = 0.40 m, Thickness of IVth material, L_4 = 85 cm = 0.85 m

Thermal conductivity of Ist material, k_1 = 50 W/m-deg,

Thermal conductivity of IInd material, k_2 = 80 W/m-deg

Thermal conductivity of IIIrd material, k_3 = 150 W/m-deg

Thermal conductivity of IVth material, k_4 = 55 W/m-deg

$A_1 = A_2 = A_3 = A_4 = 1\ m^2$

To determine: i) Rate of heat loss per unit area, Q

$$= \frac{\Delta T}{\sum R_t}$$

$$\Delta T = T_1 - T_4 = 525 - (50) = 475°C$$

$\sum R_t$ **is due to thermal resistances,** $R_{t_1}, R_{t_2}, R_{t_3}$ **and** R_{t_4}

$$R_{t_1} = \frac{L_1}{k_1 A_1} = \frac{0.25}{50 \times 1} = 0.005\ deg/W$$

$$R_{t_2} = \frac{L_2}{k_2 A_2} = \frac{0.85}{80 \times 1} = 0.011 deg/W$$

$$R_{t_3} = \frac{L_3}{k_3 A_3} = \frac{0.4}{150 \times 1} = 0.0026\ deg/W$$

$$R_{t_4} = \frac{L}{k_4 A_4} = \frac{0.85}{55 \times 1} = 0.015\ deg/W$$

The resistances R_2 and R_4 are in parallel and their equivalent resistance R_{eq} is

$$R_{eq} = \frac{R_{t2} \times R_{t4}}{R_{t2} + R_{t4}} = \frac{0.011 \times 0.015}{0.011 + 0.015} = 0.0063\ deg/W$$

Total resistance $R_t = \sum R_t = 0.005 + 0.0063 + 0.0026 =$ **0.0139 deg/W**

∴ Heat loss Q = $\frac{\Delta T}{\sum R_t} = \frac{525-(50)}{0.0139} = 34.17\ kW$

Ex 6.2 **A pipe having inner and outer diameter of 15 cm and 18 cm is carrying hot oil at temperature 200°C. The pipe is located in a room which is maintained at 20°C and the thermal conductivity of the pipe material is 250 W/(m-K). Neglecting surface heat transfer coefficients, determine the heat loss through the pipe per unit length and the temperature at a point halfway between the inner and outer surface.**

Solution:

Given: T_1 = 200°C, T_2 = 20°C, Inner diameter of pipe, d_i = 15 cm = 0.15 m,

Outer diameter of pipe, d_0 = 18 cm = 0.18 m

Thermal conductivity of pipe material, k_1 = 250 W/m-deg,

Heat transferring area, $A_i = \pi\, d_i\, L = 3.142 \times 0.15\, L$, $A_o = \pi\, d_o\, L = 3.142 \times 0.18\, L$

L is the length of pipe which is not given but can be assumed to be 1m as heat loss per meter length of pipe is to be determined.

To determine: i) Rate of heat loss per unit length, Q

$$= \frac{\Delta T}{\sum R_t}$$

$$R_t = \frac{1}{2\pi kl} log_e\, r_2/r_1 = \frac{1}{2\pi \times 250 \times 1} log_e(0.09/0.075)$$

$$= 1.16 \times 10^{-4} \text{deg/W}$$

$$\text{Heat loss Q} = \frac{\Delta t}{R_t} = \frac{200-20}{1.4516 \times 10^{-4}} = 1551\ kW$$

(ii) Temperature at halfway through the pipe, T

Radius at halfway through the pipe wall, r

$$r = \frac{r_1 + r_2}{2} = \frac{9 + 7.5}{2} = 8.25cm = 0.0825\ m$$

Thermal resistance of cylindrical pipe upto its mid-plane

$$= \frac{1}{2\pi kl} log_e(r/r_1) = \frac{1}{2\pi \times 250 \times 1} log_e(0.0825/0.075)$$

$$= \mathbf{6.06 \times 10^{-5} deg/W}$$

Since heat flow through each section is same;

$$1551 \times 10^3 = \frac{T_1 - T}{6.06 \times 10^{-5}}$$

∴ Temperature at the mid plane,

$$T = 300 - 1551000 \times 6.06 \times 10^{-5} = \mathbf{105.9°C}$$

Ex 6.3 **Determine the rate of heat loss per unit length from an insulated pipe (k = 225 W/m-deg) of 30 mm inner diameter which is having a thickness of 4 mm and is covered with 20 mm thick insulation (k= 0.05 W/m-deg). The inside and outside convective heat transfer coefficients are 10 W/m^2-deg and 5 W/m^2-deg respectively. Fluid being carried in the pipe is at 300 °C and air surrounding the pipe is at 30 °C.**

Solution:

Given: T_1=300°C, T_2=30°C, Inner pipe diameter, d_i =30 mm, r_1=15 mm =0.015 m,

Thickness of Pipe = 4 mm, Thickness of insulation = 20 mm,

Outer pipe diameter, $d_2 = d_1 + 2$ X Pipe thickness = 30 + 8 mm =38 mm

r_2=19 mm = 0.019 m, $r_s = r_2$ + thickness of insulation = 0.019 + 0.02=0.039 m

Heat transferring area:

$A_i = 2\pi\ r_i\ L$ $A_2 = 2\pi\ r_2\ L$, $A_3 = 2\pi\ r_3\ L$

Thermal conductivity of pipe material, k_1 =225 W/m-°C,

Thermal conductivity of insulation, k_2 =0.05 W/m-°C,

Inside convective heat transfer coefficient, hi = 10 W/(m^2-deg)

Outside convective heat transfer coefficient, ho = 5 W/(m^2-deg)

L is the length of pipe which is not given but can be assumed to be 1m as heat loss per meter length of pipe is to be determined.

To determine: i) Rate of heat loss per unit length, Q

$$= \frac{\Delta T}{\sum R_t}$$

The thermal resistances offered to flow of heat are as:

a. Inside fluid film,

$$R_{t_1} = \frac{1}{h_i A_i} = \frac{1}{h_i 2\pi r_1 L} = \frac{1}{10\ \times 2 \times 3.142 \times 0.015 \times 1} = \mathbf{1.06\ deg/W}$$

b. Pipe material,

$$R_{t_2} = \frac{log_e\, r_2/r_1}{2\pi k_1 L} = \frac{log_e(0.019|0.015)}{2 \times 3.142 \times 225 \times 1} = \mathbf{1.67 \times 10^{-4}\ \ deg/W}$$

c. Insulation,

$$R_{t_3} = \frac{log_e\, r_3/r_2}{2\pi k_2 L} = \frac{log_e\ (0.039|0.019)}{2 \times 3.142 \times 0.05 \times 1} = \mathbf{2.28\ deg/W}$$

d. Outside air film,

$$R_{t_4} = \frac{1}{h_0 A_0} = \frac{1}{h_0 2\pi r_3 L} = \frac{1}{5 \times 2 \times 0.039 \times 1} = \mathbf{2.56\ deg/W}$$

$$\sum R_t = R_{t1} + R_{t_2} + R_{t3} + R_{t4} = \mathbf{1.06 +\ 1.67 \times 10^{-4} + 2.28 +\ 2.56 = 5.900167\ deg/W}$$

$$\mathbf{\Delta T = T_1\ - T_2\ =\ 270^o\ C}$$

$$Q = \frac{\Delta T}{\sum R_t} = \frac{270}{5.900167} = \mathbf{45.76\ W/m}$$

HEAT GENERATION NUMERICAL

Example 6.4 An electric heater is providing uniform heat flux to a wall of 5 cm thickness having conductity, k=0.5 W/(m-deg) so that its outer surface is maintained at 15 ^oC. The outer and inner surfaces of the wall are exposed to air at 4 ^oC and 50 ^oC respectively. Inside and outside convective heat transfer coefficients are 20 W/(m^2-deg) and 80 W/(m^2-deg) . Determine the energy supplied to electric heater per unit area of the wall.

Solution:

Given: Temperature of air in contact with inner surface, T_1=50°C,

Temperature of air in contact with outer surface, T_a=4°C,

Temperaute of outer surface of wall, T_{os}=15°C,

Thickness of wall = 5 cm = 0.05 m, Conductivity of wall, k = 0.5 W/(m-deg)

Inside convective heat transfer coefficient, h_i = 20 W/(m^2-deg)

Outside convective heat transfer coefficient, h_o = 80 W/(m^2-deg)

Let q_g amount of energy is supplied to electric heater.

To determine: i) Energy supplied to electric heater per unit area of wall, q_g

From heat balance considerations for unit area of wall, it is clear that the heat gained by the outer surface from inner surface and electic heater is being lost to outside air by convection. Therefore,
Heat gained by outer surface from inside hot air + Heat supplied by electric heater, q_g = Heat lost by outer surface to surrounding air by convection

$$\frac{(T_1 - T_{os})}{\frac{1}{h_i} + \frac{\delta}{k}} + q_g = h_0(T_{os} - T_a)$$

Substituting the given data,

$$\frac{(50 - 15)}{\frac{1}{20} + \frac{0.05}{0.5}} + q_g \quad = 80 \times [15 - (4)]$$

Or $$\frac{35}{0.05+0.1} + q_g = 880$$

∴ Electric power to be provided,

$$q_g = 880 - \frac{35}{0.15} = \mathbf{646.66\ W/m^2}$$

Ex 6.5 Determine temperature at surface and maximum temperature of a 3 m long wire of diameter 5 mm when a current of 500 amperes is passing through it and it is immersed in a fluid maintained at 40 ^oC. Thermal conductivity,

resistivity and convective heat transfer coefficient of the wire are 40 W/(m-deg), 50 micro-ohm-cm and 3500 W/(m^2-deg) respectively.

Solution:

Given: Length of wire = 3 m, Diameter of wire, d = 5 mm = 0.005 m

Current, I = 500 amp, Thermal conductivity, k = 40 W/ (m-deg),

Resistivity of wire, ρ = 50 micro-ohm-cm

Convective hea transfer coefficient, h = 3500 W/(m2-deg)

Temperature of fluid, T_f=40°C,

To determine: i) Temperatue at surface of wire, T_2

$$T_2 = T_f + \frac{q_g}{2h} r$$

$$Electric\ reisitance\ of\ wire,\ R_e = \frac{\rho L}{A} = \frac{50\times10^{-6}\times300}{\frac{\pi}{4}\times(0.5)^2} = \mathbf{0.076\Omega}$$

$$Volume\ of\ wire,\ V = \frac{\pi}{4}d^2L = \frac{\pi}{4}\left(\frac{5}{1000}\right)^2 \times 3 = 5.9\times10^{-5}m^3$$

Heat generated per unit volume,

$$q_g = \frac{I^2R_e}{volume\ of\ wire} = \frac{500^2\times \mathbf{0.076}}{5.9\times10^{-5}} = 3.22\times10^8\ W/m^3$$

Radius of wire, $r\frac{5/2}{1000} = 0.0025\text{m}$

Therefore, wire surface temperature

$$T_2 = T_f + \frac{q_g}{2h}r = 40 + \frac{3.22\times10^8}{2\times3500}\times0.0025 = 40 + 115.18 = \mathbf{155.18\ °C}$$

ii) Maximum Temperature of wire, T_{max}

Maximum temperature in the wire occurs at its geometric centre line, and can be computer from the relation,

$$\boldsymbol{T_{max} = T_f + \frac{q_g}{2h}r + \frac{q_g}{4k}r^2}$$

As $\boldsymbol{T_2 = T_f + \frac{q_g}{2h}r}$, the above equation may be written as

$$T_{max} = T_2 + \frac{q_g}{4k}r^2$$

$$= 155.18 + \frac{3.22\times10^8}{4\times40}\times(0.0025)^2 = 155.18 + 12.59 = \mathbf{167.77°C}$$

Ex 6.6 **An copper wire of diameter 10 mm and having electrical resistance of 0.075 ohm/km is carrying a current of 1000 amperes. Thermal conductivity of copper is 400 W/m-K and convective heat transfer coefficient for heat**

transfer from the wire surface to the surroundings is 20 W/m^2-K. If the surrounding are at a temperature of 30°C determine for 1km length of wire

(i) Rate of heat generation per unit volume of the wire.

(ii) Maximum Temperature and Surface Temperature of copper wire

Solution

Given: Length of wire, L = 1 km= 1000 m, Diameter of wire, d = 10 mm = 0.01 m

Current, I = 1000 amp, Thermal conductivity, k = 400 W/ (m-K),

Electrical Resistance of wire, R_e = 0.075 ohm/ km

Convective hea transfer coefficient, h = 20 W/(m^2-K)

Temperature of fluid, T_f=30°C = 30+273=303

To determine: i) Rate of heat generation per unit volume of wire, q_g

$$q_g = \frac{I^2 R_e}{volume\ of\ wire} = \frac{1000^2 \times \mathbf{0.075}}{\frac{\pi}{4} \times 0.01^2 \times 1000} = \mathbf{9.5 \times 10^5}\ \boldsymbol{W/m^3}$$

ii) Surface Temperature and Maximum Temperature of wire, T_{max}

$$Surace\ Temperature, T_2 = T_f + \frac{q_g}{2h} r = 303 + \frac{9.5 \times 10^5}{2 \times 20} \times \frac{0.01}{2} = 303 + 112.5 \quad = \mathbf{415.5\ K}$$

$$\boldsymbol{T_{max} = T_f + \frac{q_g}{2h} r + \frac{q_g}{4k} r^2}$$

As $\boldsymbol{T_2 = T_f + \frac{q_g}{2h} r}$, the above equation may be written as

$$T_{max} = T_2 + \frac{q_g}{4k} r^2$$

$$= 415.5 + \frac{9.5 \times 10^5}{4 \times 400} \times (0.01/2)^2 = 415.5 + 0.0148 = \mathbf{415.51}\ \boldsymbol{K}$$

The small difference between surface and maximum temperature occuring at centre of wire is on account of high thermal conductivity of copper and small heat generation rate.

Ex 6.7 **The inner and outer surfaces of a hollow cylinder are maintained at 500 °C and 475 °C by a heat source which is generating heat at a rate of 7× $\boldsymbol{10^6 w/m^3}$. If inner and outer radii of the clinder are 2 cm and 3.5 cm respectively and thermal conductivity of the cylinder material is 20 W/m-deg, determine temperature at mid radius of the cylinder.**

Solution

Given: Inner radius of cylinder, r_i = 2 cm = 0.02 m,

Outer radius of cylinder, r_o = 3.5 cm = 0.035 m

Mid radius of cylinder, r_m = 0.027 m

Thermal conductivity, k = 20 W/ (m-deg),

Temperature of inner surface, T_1 =500°C

Temperature of outer surface, T_2 =475°C

Heat eneration rate, q_g = $7\times 10^6 w/m^3$

To determine: i) Temperatur at mid radius of cylinder

For the specified boundary conditions, the temperature distribution is given by

$$T - T_1 = \frac{q_g}{2k}(r_1^2 - r_m^2) + \left[(T_1 - T_2) - \frac{q_g}{4k}(r_2^2 - r_1^2)\right]\frac{\log_e(r_m/r_1)}{\log_e(r_1/r_2)}$$

Inserting the appropriate data,

$$T - 500 = \frac{7\times 10^6}{4\times 20}(0.02^2 - 0.027^2) + \left[(500-475) - \frac{7\times 10^6}{4\times 20}(0.035^2 - 0.02^2)\right]\frac{\log_e(0.027/0.02)}{\log_e(0.02/0.035)}$$

$$= -28.78 + (25 - 72.18)\times\frac{0.3}{(-0.56)} = -3.505$$

∴ Temperature at mid radius, T=500-3.505=**496.495°C**