LESSON 7

Thermal Insulation

The purpose of use of thermal insulation is to prevent or reduce transfer of thermal energy between two systems maintained at different temperatures. Thermal insulation is generally used in following applications

- Protective clothing for human comfort
- Design of energy efficient buildings
- Air Conditioning systems
- Refrigeration and food preservation
- Automobiles
- Boilers and steam pipes
- Spacecraft

Value of thermal conductivity of a material is generally used as a measure of its insulating capabilities. Materials having lower value of thermal conductivity are considered to be insulators. Insulating capability of a material depends upon following factors

- Thermal conductivity
- Temperature
- Density or Porosity
- Specific heat
- Surface emissivity
- Moisture Content
- Air Pressure
- Convection with in insulating material

According to 'Thermal Insulation Association of Canada' thermal insulation is used for temperature range of -75 ^{o}C and 815 ^{o}C as for temperature range below -75 ^{o}C and higher than 815 ^{o}C, cryogenic and refractory materials are used respectively. Depending upon temperature range for which thermal insulation is to be used, insulating materials are categorized as

i) Low temperature insulating materials for temperature range of -75 ^{o}C to 15 ^{o}C
ii) Medium temperature insulating materials for temperature range of 15 ^{o}C to 315 ^{o}C
iii) High temperature insulting materials for temperature range of 315 ^{o}C to 815 ^{o}C

The most commonly used insulating materials and their properties are given in Table 1

TABLE 1– Thermal Conductivities of Insulating Materials (Powders)

	Material	Mean Temperature °C	Conductivity (K) k cal/ m-hr-°C
1.	Alumina compressed powder	50	7.0
2.	Ashes, Soft wood	25	0.33
3.	Carbon black	55	0.22
4.	Coal dust	40 100	1.20 1.32
5.	Coke dust	25	1.50
6.	Charcoal	15	0.54
7.	Cork-granulated	0	0 45
8.	Floto foam	35	0.30
9.	Graphite powder	40	1.40
10.	Plaster of Paris	25	10.5
11.	Fine river sand Moistured	10	3.3
12.	Saw dust	25	11.6
13.	Silica	30	0.06
14.	Silica Aerogel	315 480 650	0.80 0.84 0.93
15.	Silica gel	-70 -20 40 100 150 55	0.174 0.207 0.240 0.276 0.318 0.90
16.	Snow	0	0.51 to 1.9

Critical Thickness of Insulation

Addition of layer insulation to walls and slabs always reduces the heat transfer, however, in case of systems having cylindrical and spherical surfaces, addition of insulation does not reduce the heat transfer, rather it increases heat transfer rate upto certain thickness of insulation. As insulation layer is added to a bare pipe, heat transfer rate increases instead of decreasing. With further addition insulation layer, heat transfer rate goes on increasing till it attains a maximum value beyond which it decreases with increase in thickness of insulation material. The thickness of insulation corresponding to maximum heat transfer rate from the pipe is called critical thickness of insulation.

This concept of critical thickness of insulation finds a useful application in electric transmission cables. Transmission losses increase with increase in temperature as electric current passes through the cables. In order to reduce the transmission losses, it becomes imperative to increase heat transfer rate from cables to surroundings so that temperature of cables is reduced. This is achieved by keeping the thickness of insulation equal to its critical thickness.

In order to determine the critical thickness of insulation, consider a cylinder of negligible thickness, length 'L', radius 'r_1' and it is carrying hot fluid at temperature T_1. Temperature of hot fluid is higher than that of ambient temperature T_a. The cylinder is insulated by an insulating material having thickness 't' and thermal conductivity 'k'. Thickness of insulation can be expressed as

$$t = r_2 - r_1 \tag{1}$$

where r_2 is the outer radius of the arrangement consisting of thin cylinder and layer of insulating material and it depends upon value of the thickness of insulating material. The arrangement has been shown in Figure 1.

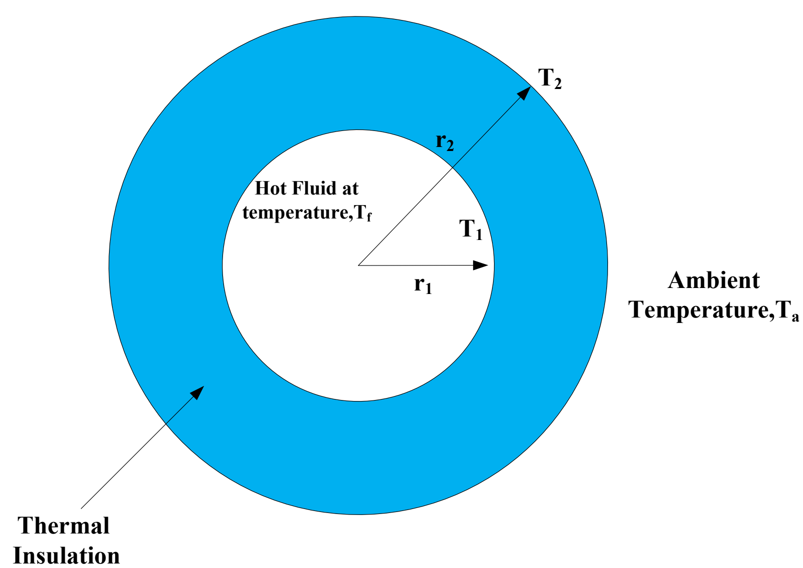

Figure 1

Heat transfer rate per unit length from cylinder is expressed as

$$Q = \frac{T_1 - T_a}{\frac{1}{2\pi r_1 L}\frac{1}{h_i} + \frac{1}{2\pi k L}\log_e\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi r_2 L}\frac{1}{h_o}} \tag{2}$$

h_i and h_o inside and outside heat transfer coefficients of convection

By using equation (1), heat transfer rate per unit length can be determined for different value of r_2. Total thermal resistance of the arrangement is sum of inside convective thermal resistance, conductive thermal resistance of insulating material and outside convective thermal resistance. Total thermal resistance of the arrangement depends upon the value of r_2 which in turn depends upon the thickness of insulating material. As the thickness of insulation increase, r_2 also increase resulting in increase in conductive thermal resistance and decrease in outer convective resistance. Therefore, increase in thickness of insulation will either result in increase or decrease in heat transfer rate depending on the overall change in the total thermal resistance.

For heat transfer rate per unit length to be maximum, thermal resistance should be the minimum. The value of r_2, for which heat transfer rate per unit length will be maximum, can be obtained by differentiating total thermal resistance i.e. denominator of equation (2) with respect to r_2 and equating equal to zero.

$$\frac{d}{dr_2}\left(\frac{1}{2\pi r_1}\frac{1}{h_i}+\frac{1}{2\pi k}\log_e\left(\frac{r_2}{r_1}\right)+\frac{1}{2\pi r_2}\frac{1}{h_o}\right)=0$$

$$\frac{1}{2\pi k}\frac{1}{r_2}-\frac{1}{2\pi h_o}\frac{1}{r_2^2}=0$$

$$\frac{1}{2\pi r_2}\left(\frac{1}{k}-\frac{1}{r_2 h_o}\right)=0$$

$$r_2=\frac{k}{h_o}=r_c \tag{3}$$

Equation (3) gives the value of thickness of insulation for which heat transfer rate per unit length is the maximum as corresponding total thermal resistance is the minimum. The value of thickness of insulation at which total thermal resistance is minimum is called critical thickness of insulation and is represented by r_c and depends upon thermal conductivity and convective heat transfer coefficient of insulating material. Addition of insulation to a bare surface will either increase or decrease the heat transfer rate per unit lenth depending upon the value of critical radius and radius of bare surface.

i) If $r_1 < r_c$, the rate of heat transfer per unit length of cylinder increases as thickness of insulation increases. The heat transfer rate goes on increasing with increase in thickness of insulation and attains a maximum value corresponding to that value of insulation of thickness for which r_2 becomes equal to r_c as shown in Figure 2 . It is due to the fact that in ranger r_l less than r_c, progressive decreases in convective resistance with increase in insulation thickness dominates over the corresponding increase in conductive resistance. The net effect is decreases in total thermal resistance and heat transfer rate per unit length increases.

ii) For the range r_l greater r_c, heat transfer rate goes on decreasing with increase in insulation thickness as shown in Figure 3. It is attributed to the fact that in range r_l greater than r_c, effect of conductive resistance dominates over that of the convective resistance resulting in increases in total thermal resistance. Therefore, heat transfer rate per unit length decreases.

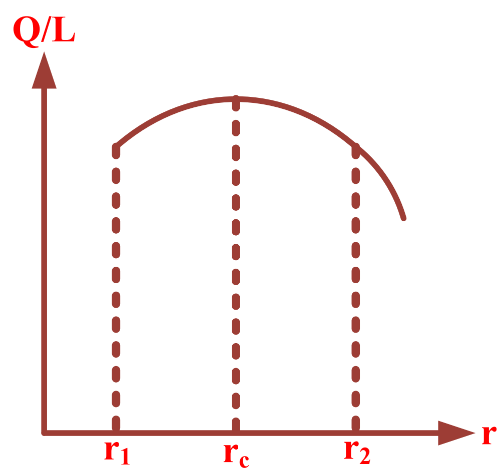

Figure 2

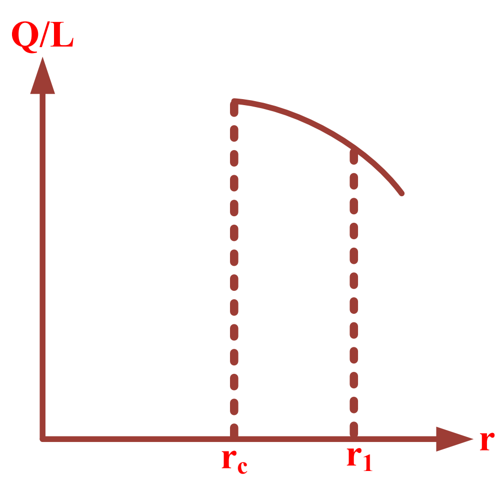

Figure 3

REVIEW QUESTIONS:

Q.1 The insulation ability of an insulator with the presence of moisture would

a) increase

b) decrease

c) remain unaffected

d) may increase/decrease depending on temperature and thickness of insulation

e) none of the above

Q.2 The critical radius of insulation for a cylindrical pipe is

a) $\frac{(\text{Thermal conductivity of insulating material})}{2 \times (\text{heat transfer coefficient at outer surface})}$

b) $\frac{2 \times (\text{Thermal conductivity of insulating material}}{(\text{heat transfer coefficient at outer surface})}$

c) Inverse of (a)

d) $\frac{(\text{Thermal conductivity of insulating material})}{(\text{heat transfer coefficient at outer surface})}$

e) Inverse of (b)

Q.3 The critical radius of insulation for a spherical shell is

a) $\frac{\text{Thermal conductivity of insulating material}}{\text{heat transfer coefficient at outer surface}}$

b) $\frac{2 \times (\text{Thermal conductivity of insulating material})}{(\text{heat transfer coefficient at outer surface})}$

c) Inverse of (a)

d) Inverse of (b)

e) none of the above

Q.4 A thin cylinder of radius r is lagged to an outer r_0 with an insulating layer of thermal conductivity k. If h_0 is the film coefficient at the outer surface of lagging, then minimum resistance and consequently maximum heat flow rate occurs when r_c equals

a) $\sqrt{kh_0}$

b) $\frac{k}{h_0}$

c) $\frac{2k}{h_0}$

d) $\frac{h_0}{k}$

Q.5. Critical thickness of insulation for spheres is given by

a) k/h

b) $k/4\pi h$

c) $h/2k$

d) $2k/h$

Where k is the coefficient of thermal conductivity and h is convective heat transfer coefficient

Q.6 What happens when the thickness of insulation on a pipe exceeds the critical value?

a) There is decrease in the heat flow rate

b) There is increase in the heat flow rate

c) The heat flows remains constant
d) The temperature at the junction between pipe and insulation rises

Q.7 Which of the following is a wrong statement?

a) Addition of insulation does not always bring about a decrease in the heat transfer rate for geometries with non-constant cross-sectional area
b) Rubber insulated wires can carry more current than a bare wire for the same rise in temperature
c) A certain thickness of lagging a stream pipe may increase the rate of heat flow rather than reduce it
d) **Critical radius of insulation refers to the outer radius of insulation for which there is maximum thermal resistance and consequently maximum heat flow rate**

SOLVED EXAMPLES ON INSULATION

Ex 7.1 In order to increase heat transfer from an electric cable of diameter 15 mm, it is provided with a plastic cover having thermal conductivity, k= 0.095 W/m-K and convective heat transfer coefficient, h= 12 W/m^2-K. Determine the percentae increase in heat transfer per unit length from cable if its surface is maintained at 85 ^{o}C and temperature of surroundings is 30 ^{o}C.

Solution:

Given: T_1= 85 °C, T_2= 30°C, Diameter of cable, d_1 = 15 mm ,

Radius of cable, r_1 = 7.5 mm= 0.0075 m

Thermal conductivity, k = 0.095 W/m-K,

Convective heat transfer coefficient, h_o = 12 W/m^2-K

L is the length of pipe which is not given but can be assumed to be 1m as heat loss per meter length of cable is to be determined.

Heat transferring area of cable, A = πdL= 3.142X0.015X1=0.0471 m^2

To determine: i) Percentage increase in heat transfer per unit length,

$$= \frac{Q_{insulated} - Q_{bare}}{Q_{bare}} \times 100$$

$$Q_{bare} = ho\,A\,(T_1 - T_2) = 12 \times 0.0471\,(85 - 30) = \mathbf{31.01\ W}$$

Maximum heat transfer from an insualted cylinderical object will occur when thickness of insulation is equal to critical thickness and corresponding to critical thickness, critical radis of insulation can be determined by using following equation

$$r_c = \frac{k}{h_o} = \frac{0.095}{5} = \mathbf{0.019\ m = 19\ mm}$$

Maximum heat transfer from the cable is given by

$$Q_{insulated} = \frac{2\pi L(T_1 - T_2)}{\frac{1}{k}\log_e\left(^{r_c}/_{r_1}\right) + \frac{1}{h_o r_c}} = \frac{2 \times 3.142 \times 1(85 - 30)}{\frac{1}{0.095}\log_e\left(^{0.019}/_{0.015}\right) + \frac{1}{12 \times 0.015}}$$

$$= \frac{345.62}{10.52\ \times 0.236 + 5.55} = \frac{345.62}{8.032} = \mathbf{43.02\ W}$$

Therefore, percentage increase in heat transfer from the cable is

$$= \frac{43.02 - 31.01}{31.01} \times 100 = \mathbf{38.75\%}$$

Ex 7.2 **An insulation of 2 mm thickness has been provided on a copper wire of diameter 2 mm. The cable is laid in an environment which is at temperature 28 °C. As a safety measure, it is required that temperature of insulation should not exceed 75 °C. Determine the maximum current carried by the copper wire if electric resistance of copper is 0.0254 ohm, thermal conductivity of copper is 400 W/m-K, thermal conductivity of insulation is 0.01 W/m-K and convective heat transfer coefficient between insulation and surroundings is 10 W/m^2 -K.**

Solution:

Given: T_1= 75 °C, T_2= 28°C, Diameter of copper wire, d_1 = 2 mm

Radius of cable, r_1 = 1 mm= 0.001 m,

Thickness of insulation, t = 2 mm =0.002 m

Radius of the arrangment consisting of wire and insulation, $r_2 = r_1 + t$,

r_2 = 0.001+0.002 = 0.003 m

Thermal conductivity of copper wire, k_c = 400 W/m-K,

Thermal conductivity of insulation, k_{in} = 0.1 W/m-K

Convective heat transfer coefficient, h_o = 10 W/m^2-K

Electric reistance of copper wire, R = 0.0254 ohm

L is the length of wire which is not given but can be assumed to be 1m.

To determine: i) Current carrying capacity of copper wire, I

$$\textit{Heat dissipated by wire is given by, } Q = I^2 R, \qquad I = \sqrt{\frac{Q}{R}}$$

$$Where, Q = \frac{2\pi L(T_1 - T_2)}{\frac{1}{k_{in}}\log_e({r_2}/{r_1}) + \frac{1}{h_o r_2}} = \frac{2 \times \pi \times 1 \times (75 - 28)}{\frac{1}{0.1}\log_e({0.003}/{0.001}) + \frac{1}{10 \times 0.003}}$$

$$Q = \frac{295.34}{10.98 + 33.33} = 6.66\ W$$

$$Therefore, 6.66 = I^2 \times 0.254,\ \ I^2 = 262.4$$

I= 16.19 amp

Ex 7.3 A pipe of diameter 2.5 cm is insultaed with 0.5 cm thick plastic having thermal conductivity of 0.4 W/m-K and convective heat transfer coefficient is 10 W/m²-K. Determine the effectiveness of the insulation if temperature at the surface of bare pipe is 100 °C and that of surroundings is 30 °C. In order to reduce the heat loss from the pipe by 40%, plactic insulation is replaced by a rubber insulation having thermal conductivity of 0.04 W/m-K. Determine the thickness of rubber pipe.

Solution:

Given: T_1= 100 °C, T_2= 30°C, Diameter of bare pipe, d_1 = 2.5 cm = 0.025 m

Radius of pipe, r_1 = 0.0125 m,

Thickness of insulation, t = 0.5 cm = 0.005 m

Radius of arrangement consisting of pipe and insulation, r_2= r_1+t

r_2 = 0.0125 +0.005 = 0.013 m

Thermal conductivity of plastic insulation, k_p = 0.4 W/m-K,

Thermal conductivity of rubber insulation, k_r = 0.04 W/m-K,

Convective heat transfer coefficient, h_o = 10 W/m²-K

To determine: i) Effectiveness of plastic insulation

ii) Thickness of rubber insulation

i) Effectiveness of plastic insulation

Plastic insulation will be effective if critical radius is more that the radius of arrangement consisting of pipe and plastic insualtion.

Critical radius $r_c = k/h_o$ = 0.4/10 =0.04 m

Radius of arrangement, r_2 =0.0013 m

Since r_c is greater than r_2, therefore addition of plastic insualtion of thickness 0.5 cm will increase the heat transfer from the pipe and will not be effective in decreasing the heat transfer.

ii) Thickness of rubber insulation

Thickness of rubber insulation is to be determined when heat loss is to be reduced by 40% of that of bare pipe.

Heat loss from bare pipe per meter length

$$Q = 2\pi \times r_1 \times L \times h_o \times (T_1 - T_2) = 2 \times 3.142 \times 0.0125 \times 1 \times 10 \times (100 - 30)$$
$$= 54.98\ W$$

By replacing plastic insulation with rubber insulation, heat loss from bare pipe, Q is to be reduced by 40%.

Q_1 = 40% of Q = 0.4 X54.98 =21.99≈ 22 W

$$Q_1 = \frac{2\pi \times (T_1 - T_2)}{\frac{1}{k_r}\log_e({r_2}/{r_1}) + \frac{1}{h_o r_2}}$$

$$22 = \frac{2 \times \pi \times 1 \times (100 - 30)}{\frac{1}{0.04}\log_e({r_2}/{0.0125}) + \frac{1}{10 \times r_2}}$$

$$22 = \frac{439.88}{25 \times \log_e({r_2}/{0.0125}) + \frac{1}{10 \times r_2}}$$

$$25 \times \log_e({r_2}/{0.0125}) + \frac{1}{10 \times r_2} = \frac{439.88}{22} = 19.99$$

By solving aboe equation by hit and trail method, we get r_2 =0.0692 m = 6.92 cm

Therefore, thickness of rubber insulation, t = $r_2 - r_1$ = 6.92-1.25 = 5.67 cm