LESSON 8

HEAT TRANSFER FROM EXTENDED SURFACES

Fins: Heat transfer from a hot surface to surrounding fluid takes place by convection and is governed by Newton's law of cooling

$$Q = h\,A\,(T_s - T_f) \qquad (1)$$

It is obvious from the above equation that heat transfer rate can be enhanced

- Either by increasing convective heat transfer coefficient, h.
- Or by increasing area of heat transferring surface, A.
- Or by increasing temperature difference, $(T_s - T_f)$

Heat transfer rate is generally enhanced by increasing area of heat transferring surface as shown in Figure 1. It is not always possible to increase the value of convective heat transfer coefficient, h by increasing velocity of flow of fluid surrounding the hot surface or to increase temperature difference by lowering temperature of fluid which is in contact with the hot surface.

Increase in heat transfer area is achieved by attaching protrusions to hot solid surface. These protrusions are referred as fins or spines and are made of materials having high thermal conductivity such as aluminum. Fins bring about considerable enhancement in heat transfer rate from a solid surface by exposing a larger surface area for convective and radiative heat transfer.

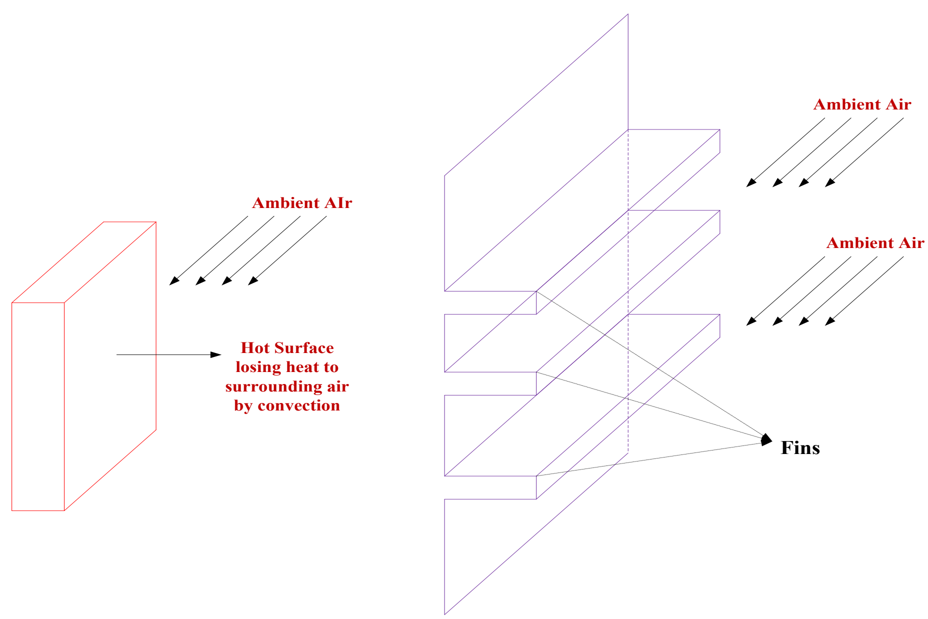

Figure 1

Fin Applications:- Fins are generally used when convective heat transfer coefficient is low such as in case of gases and under natural convection. There are numerous appliances used by us in daily life in which fins have been used to enhance heat transfer rate.

- Electrical apparatus like transformers and motors
- Engines of scooters, motorcycles and compressors The circumferential fins of rectangular or triangular profile are commonly used on the engine cylinders of scooters and motor cycles
- Condenser and evaporators of a refrigerating systems
- Car radiator

Types of Fins:

1. Straight Fin: The terms straight fin is applied to the extended surface attached to a wall which is otherwise plane. Figures 2 (a) and (b) are of fins of uniform area whereas Figures (c) and (d) are of fins of non-uniform area.

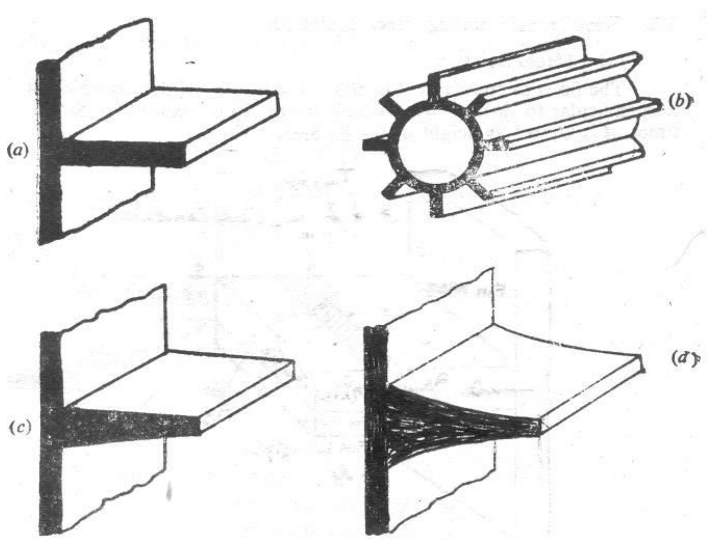

Figure 2

2. Annular (circumferential) fin: Annular fin is one, attached circumferentially, to a cylindrical surface as showon in Figure 3.

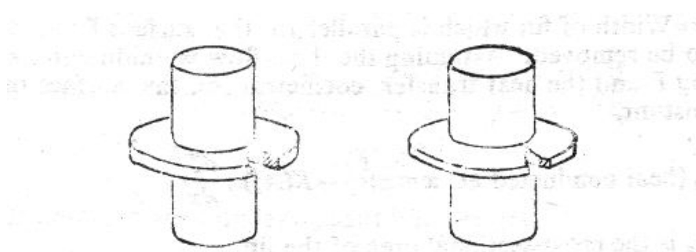

Figure 3

3. **Spine or pin fin:** It is an extended surface of cylindrical or conical shape as shown in Figure 4.

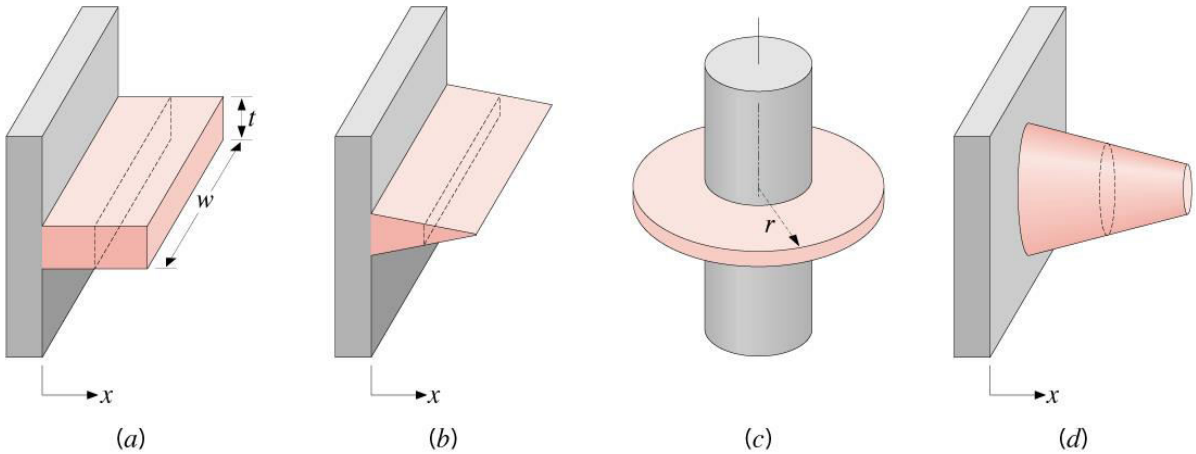

Figure 4

Heat transfer through a fin of uniform cross-section

The purpose of heat transfer analysis through a fin is to determine the increase in heat transfer obtained by attaching fins to a surface. In order to increase heat transfer from a plane wall, a rectangular fin of length 'L', width 'w' and thickness '2δ' has been attached to the wall as shown in Figure 5. Heat flows along the length of the fin and fin is surrounded by atmospheric air at temperature T_a. Temperature at the base or root of fin adjoining the wall is T_o.

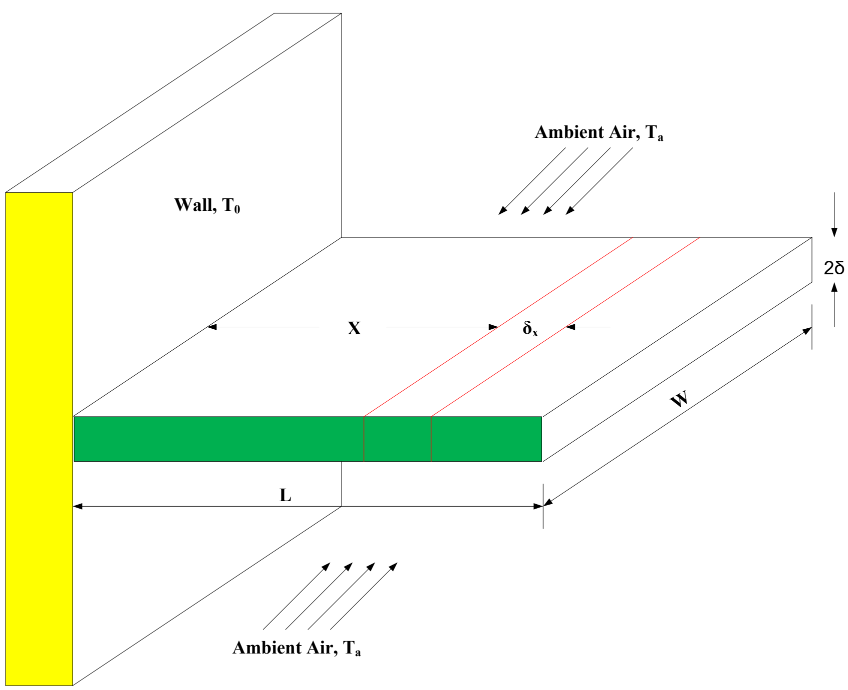

Figure 5

In order to simplify the analysis, following assumptions have been made

- Flow of heat takes place under steady state conditions and is one dimensional i.e. along the length of fin.
- Thermal conductivity of fin material is constant.
- Convective heat transfer coefficient 'h' is uniform over the entire surface of fin.
- Temperature, T_o, at the base or root of fin is uniform and is equal to wall temperature T_w.

Consider a small element of thickness 'δ_x' at a distance 'x' from the base of fin and its perimeter and cross-sectional area are expressed as

Perimeter, $P = 2w + 4\delta_x = 2(w + 2\delta_x)$

Cross-sectional area, $A_c = w \cdot \delta_x$

Surface Area, $A = P\,\delta_x$

Under steady state conditions, energy balance on this element can be expressed as

Rate of heat conducted in to the element at x	=	Rate of heat conducted from the element at x+ δ_x	+	Rate of heat convected from the element

Heat conducted into the element = $Q_x = -k A_c \dfrac{dT}{dx}\Big|_{x=x}$ (2)

Heat conducted out of the element = $Q_{x+\partial x} = Q_x + \dfrac{d}{dx}(Q_x)\,\delta_x = -k A_c \dfrac{dT}{dx}\Big|_{x=x+\delta x}$ (3)

Heat convected out of the element = $Q_c = h\,A\,(T - T_a) = h\,\delta_x\,P\,(T - T_a)$ (4)

Heat balance for the element is given by-

$$Q_x = Q_{x+\partial x} + Q_c \qquad (5)$$

Using equations (2), (3), (4), equation (5) can be written as

$$-k A_c \frac{dT}{dx}\Big|_{x=x} = -k A_c \frac{dT}{dx}\Big|_{x=x+\delta x} + h\,\delta_x\,P(T - T_a) \qquad (6)$$

$$-k A_c \frac{dT}{dx}\Big|_{x=x} = -k A_c \frac{dT}{dx}\Big|_{x=x} + \frac{d}{dx}(Q_x)\,\delta_x + h\,\delta_x\,P(T - T_a)$$

$$\frac{d}{dx}\left(-k A_c \cdot \frac{dT}{dx} \cdot \delta_x\right) = -\,h\,\delta_x\,P(T - T_a)$$

$$- k A_c \delta_x \frac{d^2T}{dx^2} = - h \delta_x P(T - T_a)$$

$$k A_c \frac{d^2T}{dx^2} = h P (T - T_a) \quad (7)$$

Let $T - T_a = \theta$ (8)

and $T_o - T_a = \theta_o$ (9)

Differentiating equation (8) with respect to 'x' twice, we get

$$\frac{dT}{dx} = \frac{d\theta}{dx}, \Rightarrow \frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2} \quad (10)$$

Using equation (10), equation (7) can be written as

$$k A_c \frac{d^2\theta}{dx^2} = h P \theta$$

$$\Rightarrow \frac{d^2\theta}{dx^2} = \frac{hP}{kA_c} \theta \quad \Rightarrow \quad \frac{d^2\theta}{dx^2} = m^2 \theta$$

where $m^2 = \frac{hP}{kA_c}$ or $m = \sqrt{\frac{hP}{kA_c}}$

$$\Rightarrow \frac{d^2\theta}{dx^2} - m^2 \theta = 0 \quad (11)$$

Equation (11) is the governing equation and its solution is expressed as

$$\theta = A e^{mx} + B e^{-mx}$$

$$\because e^{mx} = \cosh mx + \sinh mx; \quad e^{-mx} = \cosh mx - \sinh mx$$

$$\therefore \theta = (A+B) \cosh mx + (A-B) \sinh mx$$

$$\Rightarrow \theta = c_1 \cosh mx + c_2 \sinh mx \quad (12)$$

Where $C_1 = A+B$ and $C_2 = A-B$

REVIEW QUESTIONS:

1. Fins are provided on heat transferring surface in order to increase
   a) **Heat transfer area**
   b) Heat transfer coefficient
   c) Temperature gradient
   d) Mechanical strength to the equipment

2. Fins are usually provided to a heat exchanger surface in order to augment heat transfer by increasing the
   a) Heat transfer coefficient
   b) **Surface area**
   c) Turbulence level
   d) Temperature difference

3. Finned surfaces have improved rate of dissipation due to
   a) Decrease in ambient temperature
   b) **Increase in the surface area exposed to the surrounding**
   c) Increase in the convective film coefficient
   d) All of the above

4. Three fins of equal length and diameter but made of aluminium, brass and cast iron are heated to 200°C at one end. If the fins dissipate heat to the surrounding air at 25°C, the temperature at the free end will be least in case of
   a) **Aluminium fin**
   b) Brass fin
   c) Cast iron fin
   d) Each fin will have the same temperature at the free end

5. A fin of length l protrudes from a surface held at temperature t_0 ; it being higher than the ambient temperature t_a . The heat dissipation from the free end of the fin is stated to be negligibly small. The temperature gradient at the fin tip $(dt/dx)_{x=l}$ can then be expressed as

   a) 0 b) $\frac{t_0 - t_l}{l}$

   c) $h\,(t_0 - t_l)$ d) $\frac{t_l - t_a}{t_0 - t_a}$

6. A fin protrudes from a surface which is held at a temperature higher than that of its environments. The heat transferred away from the fin is
   a) Heat escaping from the tip of the fin
   b) Heat conducted along the fin length
   c) **Convective heat transfer from the fin surface**
   d) Sum of heat conducted along the fin length and that convected from the surface

7. A straight fin of cross-sectional area A for all along its length and made of a material of thermal conductivity k serves to dissipate heat to the surroundings from a surface held at a constant temperature. What additional data is required to workout the rate of heat dissipation?

a) The root and the tip temperatures
b) The temperature gradient at the root
c) The temperature gradient at the tip
d) The convective heat transfer coefficient and the fin perimeter.

8. In a particular heat transferring situation, a cast iron fin has been replaced by a copper fin of identical configuration. If all other parameters are maintained constant, such a replacement will

a) increase the total heat flow
b) decrease the total heat flow
c) heat flow is influenced only if the base temperature and sectional area
d) will effect only the temperature distribution